STUDY REGARDING THE MARKOWITZ MODEL OF PORTFOLIO SELECTION

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Abstract

The Markowitz model was introduced through the work of Harry Markowitz (1952) and analyzes the risk and the rentability of a diversified portfolio of securities. In our research, we want to use the Markowitz model in order to identify the structure of the optimal portfolio of risky assets, in other words the efficient portofolio. The study, conducted on three romanian companies from the construction sector, listed on the Bucharest Stock Exchange, leads to the conclusion that the portfolio is illegitimate, so it is necessary to sell the securities of the companies in the absence, procedure known as short sell.

Key words: *efficient portfolio, risk, rentability, scenario.*

Jel codes: *G11, G17*

1. Introduction

The Markowitz model is very useful for the investors, because it gives them the possibility to determine the optimal portfolio structure. In order to identify the investment alternatives, the next steps will be taken into consideration: to identify the portfolio with absolute minimum variation; to calculate the weights of the securities from the portfolio; to classify the portfolios in legitimate and illegitimate. Thereby, it is desirable to determine the most appropriate investment solution which has as coordinates the rentability and the risk of the portfolio.

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2. Literature review

In 1952, Harry Markowitz created a portfolio selection model in order to determine the efficient portfolios, receiving the Nobel Prize for this research in 1990 (Markowitz, Harry M., 1991) [7]. This model is based on the rentability and the risk of the diversified portfolio and involves the correlation of the portfolio securities, in order to identify the portfolio with absolute minimum variation. Markowitz starts from the idea that combining more titles in the portfolio contributes to higher rentabilities for the risk that the investor is willing to assume (Markowitz, Harry M., 1952) [4].

William Sharpe extended Harry Markowitz's research through its diagonal model of portfolio selection (1964), in order to simplify the securities selection in the portfolio (Sharpe William F., 1964) [9]. Applying the model on a portfolio, he demonstrated that the investor can control the portfolio risk trying to maximize the relationship between risk and rentability. If in his model, Markowitz groups the securities two by two and sets links between them, the model developed by William Sharpe considers necessary a relationship between the title and a macroeconomic factor.

At least two conditions must be fulfilled in order to study the securities evolution: the investors willingness to maximize the portfolios expected returns; it is necessary that the expected return to be maximum and the related risk to be minimum.

The portfolio's risk can be minimized through the Lagrange combined function (http://www.dofin.ase.ro/Lectures/Altar%20Moisa/Teoria%20portofoliului.pdf):

$$L = \frac{1}{2}\sum_{k=1}^{n} x_k \sum_{j=1}^{n} x_j \sigma_{kj} - \lambda_1\left(\sum_{k=1}^{n} x_k \mu_k - \rho\right) - \lambda_2\left(\sum_{k=1}^{n} x_k - 1\right)$$

where

xk, xj – the weight of the titles "k" and "j" in the portfolio
s kj – the covariance between the title "k"and the title "j"
μk – the expected return of the title "k"
P – the expected return of the portfolio
λ1, λ2 – the Lagrange multipliers concerning the two restrictions

According to the Markowitz model, the efficient portfolios who have also negative weights of titles are called illegitimate. Initially, the model

required that the titles weights to be positive. Later, in the diagonal model, Sharpe relaxed this restriction, being accepted the fact that, hardly ever, there are titles sold on debit (in absence), procedure known as short sale. These types of efficient portfolios push the market to practice another interest rates than those which result from the supply and the demand for money, and for this reason are called illegitimate (Sharpe, William F., 1963) [8].

3. The research methodology

The research conducted is based on the financial information provided by three major companies from the romanian construction sector, listed on the Bucharest Stock Exchange (www.bvb.ro): Impact Developer & Contractor S.A. (IMP), Transilvania Construcţii S.A. (COTR) and Condmag S.A. (COMI), over a period of 6 months (01.05.2014 - 02.10.2014), and supposed the following steps:

- the calculation of the weekly rentabilities of the shares of the three companies, taking into account the weekly closing price of the securities;
- the calculation of the Markowitz matrix through the COVAR function from EXCEL, in order to determine the covariances between the weekly closing prices for each company;
- the determination of the Markowitz's inverse matrix using the MINVERSE function;
- the Markowitz's inverse matrix was multiplied with the K matrices (K1, K2, K3, K4, K5) using the MMULT function, in order to determine the X matrices. The K matrices received random values of the expected return (E*p), in order to form 5 different scenarios;
- the X matrices provide information about the portfolio structure, ie the percentage which is recommended to be invested in each company in order to obtain an efficient portfolio; when the obtained weights are negative, the portfolio is considered to be illegitimate and it is recommended to sell the securities.

3.1. The determination of the titles weekly return

In order to apply the Markowitz model, we determined the average weekly return of the analyzed companies, based on the weekly closing prices of the reference period (table nr.1).

Table 1: Own processing of the data provided by the portal www.bvb.ro

Data	IMP		COTR		COMI	
	Pi		**Pi**		**Pi**	
01.05	0.79		34.1700		0.0715	
08.05	0.77	-2.5316	37.4900	9.7161	0.0719	0.5594
15.05	0.77	0	50.2500	34.0357	0.0678	-5.7024
22.05	0.77	0	43.9800	-12.477	0.0669	-1.3274
29.05	0.74	-3.8961	41.8400	-4.8658	0.0687	2.6906
05.06	0.71	-4.0541	48.6300	16.2285	0.0702	2.1834
12.06	0.75	5.6338	51.1000	5.0792	0.0703	0.1425
19.06	0.8155	8.7333	51.4750	0.7339	0.069	-1.8492
26.06	0.985	20.7848	51.9000	0.8256	0.066	-4.3478
03.07	0.99	0.5076	51.9500	0.0963	0.068	3.0303
10.07	0.97	-2.0202	51.4750	-2.0202	0.0661	-2.7941
17.07	0.97	0	51.0000	-0.9228	0.067	1.3616
24.07	1.003	3.4021	51.4750	0.9314	0.0674	0.597
31.07	1.0500	4.6859	51.0000	-0.9228	0.0679	0.7418
07.08	1.00	-4.7619	47.1750	-7.5	0.0675	-0.5891
14.08	1.00	0	47.1750	0	0.0675	0
21.08	1.00	0	43.3500	-8.1081	0.0647	-4.1481
28.08	1.0300	3	43.7000	0.8074	0.0639	-1.2365
04.09	1.1900	15.534	43.7000	0	0.0588	-7.9812
11.09	1.3500	28.5714	43.3500	-0.8009	0.0624	6.1224
18.09	1.3490	-0.0741	43.3500	0	0.0607	-2.7244
25.09	1.3000	-3.6323	43.3500	0	0.0599	-1.318
02.10	1.3600	4.6154	45.0000	3.8062	0.0594	-0.8347
		74.498		34.6421		-17.4239
		Rs1m = 3.3863		Rs2m = 1.5746		Rs3m = -0.7920

Source: The data was processed by the authors, based on the annual financial statements, available on the webite www.bvb.ro

The IMP company has the highest weekly return during the analyzed period, respectively 3,39%, followed by the COTR company which recorded a 1,57% weekly return. Instead, the COMI company's shares recorded a loss of 0,79%.

3.2. The determination of the Markowitz matrix (W)

The correlated evolution of the shares price of the three companies, grouped two by two, during the analyzed period is presented in the table nr.2.

Figure 1: The system of Markowitz matrix equations for the portfolio of 3 titles

$$\begin{bmatrix} \sigma 1^2 & \sigma 12 & \sigma 13 & E1 & 1 \\ \sigma 21 & \sigma 2^2 & \sigma 23 & E2 & 1 \\ \sigma 31 & \sigma 32 & \sigma 3^2 & E3 & 1 \\ E1 & E2 & E3 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} X1 \\ X2 \\ X3 \\ \lambda 1 \\ \lambda 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ E^*p \\ 1 \end{bmatrix}$$

W x X = K

Source:http://www.dofin.ase.ro/Lectures/Altar%20Moisa/Teoria%20portofoliului.pdf)

where

s_{12} = the covariance between the title 1 and the title 2, respectively the title of the IMP company and that of the COTR company;

s_{13} = the covariance between the title 1 and the title 3, respectively the title of the IMP company and that of the COMI company;

s_{23} = the covariance between the title 2 and the title 3, respectively the title of the COTR company and that of the COMI company;

E_1, E_2, E_3 = the average weekly returns of the titles IMP, COTR and COMI.

Table 2: The determination of the Markowitz matrix

Cov IMP-IMP 376.27368	Cov IMP-COTR 162.6420031	Cov IMP-COMI -65.02303981	EIMP 3.3863	1

Cov COTR-IMP 153.1010672	Cov COTR-COTR 82.36704409	Cov COTR-COMI -33.34798571	ECOTR 1.5746	1
Cov COMI-IMP -61.45664804	Cov COMI-COTR -31.31104933	Cov COMI-COMI 24.20505753	ECOMI -0.792	1
EIMP 3.3863	ECOTR 1.5746	ECOMI -0.792	0	0
1	1	1	0	0

Source: The data was processed by the authors, based on the annual financial statements, available on the webite www.bvb.ro

It can be noticed that between the values of the three titles, grouped two by two, there are positive and negative covariances. The positive covariances means that the titles yields have the tendency to evolve in the same direction. Instead, the negative covariances reveal the titles yields tendency to evolve in opposite directions. Thus, it can be noticed that between the titles of the IMP company and those of the COTR company there is a positive covariance, therefore their yields will evolve in the same direction. Instead, between the titles of the IMP company and those of the COMI company, and between the titles of the COTR company and those of the COMI company there is a negative covariance, and their yields will evolve in opposite directions.

3.3. The determination of the Markowitz's inverse matrix

It is determined the Markowitz's inverse matrix, which multiplied by the column vector K will generate the X matrix (table nr. 3).

Table 3: The determination of the Markowitz's inverse matrix

0.012450547	-0.02198179	0.00953125	0.100654041	-0.07446268
-0.02198179	0.038809491	-0.01682769	0.244839525	0.466123319
0.00953125	-0.01682769	0.007296444	-0.34549356	0.608339368
0.158894736	0.142013912	-0.30090864	-29.2880916	-0.84496076
-0.02796406	0.38402867	0.643935395	-0.98143450	-5.37551526

Source: The data was processed by the authors, based on the annual financial statements, available on the webite www.bvb.ro

3.4. The column vector K

In order to form the column vector K, this one received random values of the expected return (E*p), respectively 20%, 30%, 40%, 50% and 60%, in order to form 5 different scenarios (table nr. 4).

Tabel 4: The K1-K5 vectors according to the 5 scenarios

K1 vector	K2 vector	K3 vector	K4 vector	K5 vector
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0.2	0.3	0.4	0.5	0.6
1	1	1	1	1

Source: The data was processed by the authors, based on the annual financial statements, available on the webite www.bvb.ro

The construction of the matrices X1-X5 has as purpose to calculate the weights recommended to be invested in each of the three companies in order to obtain the efficient portfolio, based on the expected returns of 20%, 30%, 40%, 50% and 60% (table nr.5).

3.5. The calculation of the weights recommended to be invested in each of the three companies in order to obtain the efficient portfolio, based on the expected returns

3.5.1. The X1 matrix

The construction of the X1 matrix has as purpose to calculate the weights recommended to be invested in each of the three companies in order to obtain the efficient portfolio, based on an expected return of 20% (table nr.5).

Table 5: The determination of the X1 matrix

IMP	-0.0543291	-0.0543291	-0.0543291	-0.0543291	-0.054329
COTR	0.5150909	0.5150909	0.5150909	0.5150909	0.5150909
COMI	0.5392406	0.5392406	0.5392406	0.5392406	0.5392406
	-6.7025794	-6.7025794	-6.7025794	-6.7025794	-6.7025794
	-5.5718021	-5.5718021	-5.5718021	-5.5718021	-5.5718021

Source: The data was processed by the authors, based on the annual financial statements, available on the webite www.bvb.ro

The portfolio has also negative weights of titles, being considered an illegitimate portfolio. In the case of a potential investment in the portfolio, considering that it has an expected return of 20%, it is recommended to allocate 51,51% from the available amount in the shares of the COTR company and 53,92% in the shares of the COMI company. It is recommended that 5,43% of the shares of the IMP company to be sold through the procedure of "short sale"; thus, the company could increase its investment budget, selling securities that it does not own and that can be redeemed at their rate of expected return. This operation is not regulated on the romanian capital market.

3.5.2. The X2 matrix

The construction of the X2 matrix has as purpose to calculate the weights recommended to be invested in each of the three companies in order to obtain the efficient portfolio, based on an expected return of 30% (table nr.6).

Table 6: The determination of the X2 matrix

IMP	-0.0442637	-0.0442637	-0.0442637	-0.0442637	-0.044263
COTR	0.5395748	0.5395748	0.5395748	0.5395748	0.5395748
COMI	0.5046913	0.5046913	0.5046913	0.5046913	0.5046913
	-9.631388	-9.6313882	-9.6313882	-9.6313882	-9.6313882
	-5.6699456	-5.6699456	-5.6699456	-5.6699456	-5.6699456

Source: The data was processed by the authors, based on the annual financial statements, available on the webite www.bvb.ro

The obtained portfolio is illegitimate. In the case of a potential investment in the portfolio, considering that it has an expected return of 30%, it is recommended to allocate 53,96% from the available amount in the shares of the COTR company and 50,47% in the shares of the COMI company. It is necessary that 4.42% of the shares of the IMP company to be sold through the

procedure of "short sale", operation which, as it was mentioned before, it can not be realized on the romanian capital market.

3.5.3. The X3 matrix

The construction of the X3 matrix has as purpose to calculate the weights recommended to be invested in each of the three companies in order to obtain the efficient portfolio, based on an expected return of 40% (table nr.7).

Tabelul 7: The determination of the X3 matrix

IMP	-0.03419838	-0.03419838	-0.03419838	-0.03419838	-0.034198384
COTR	0.56405881	0.56405881	0.56405881	0.56405881	0.56405881
COMI	0.470141944	0.470141944	0.470141944	0.470141944	0.470141944
	-12.5601974	-12.5601974	-12.5601974	-12.5601974	-12.5601974
	-5.76808906	-5.76808906	-5.76808906	-5.76808906	-5.76808906

Source: The data was processed by the authors, based on the annual financial statements, available on the webite www.bvb.ro

The portfolio is illegitimate. In the case of a potential investment in the portfolio, considering that its expected return is 40%, it is recommended to allocate 56,41% from the available amount in the shares of the COTR company and 47,01% in the shares of the COMI company. It is necessary that 3,42% of the shares of the IMP company to be sold through the procedure of "short sale", operation which is not regulated on the romanian capital market.

3.5.4. The X4 matrix

The construction of the X4 matrix has as purpose to calculate the weights recommended to be invested in each of the three companies in order to obtain the efficient portfolio, based on an expected return of 50% (table nr.8).

Table 8: The determination of the X4 matrix

IMP	-0.02413298	-0.02413298	-0.02413298	-0.02413298	-0.02413298
COTR	0.58854276	0.58854276	0.58854276	0.58854276	0.58854276

COMI	0.43559258	0.43559258	0.43559258	0.43559258	0.43559258
	-15.4890065	-15.4890065	-15.4890065	-15.4890065	-15.4890065
	-5.86623252	-5.86623252	-5.86623252	-5.86623252	-5.86623252

Source: The data was processed by the authors, based on the annual financial statements, available on the webite www.bvb.ro

The portfolio is illegitimate. In the case of a potential investment in the portfolio, considering that its expected return is 50%, it is recommended to allocate 58,85% from the available amount in the shares of the COTR company and 43,55% in the shares of the COMI company. It is necessary that 2,41% of the shares of the IMP company to be sold, without actually holding the share (short sale).

3.5.5. The X5 matrix

The construction of the X5 matrix has as purpose to calculate the weights recommended to be invested in each of the three companies in order to obtain the efficient portfolio, based on an expected return of 60% (table nr.9).

Table 9: The determination of the X5 matrix

IMP	-0.01406757	-0.01406757	-0.01406757	-0.01406757	-0.01406757
COTR	0.61302671	0.61302671	0.61302671	0.61302671	0.61302671
COMI	0.40104323	0.40104323	0.40104323	0.40104323	0.40104323
	-18.4178157	-18.4178157	-18.4178157	-18.4178157	-18.4178157
	-5.96437597	-5.96437597	-5.96437597	-5.96437597	-5.96437597

Source: The data was processed by the authors, based on the annual financial statements, available on the webite www.bvb.ro

The portfolio is illegitimate. In the case of a potential investment in the portfolio, considering that its expected return is 60%, it is recommended to allocate 61,30% from the available amount in the shares of the COTR company and 40,10% in the shares of the COMI company. It is necessary that 1,41% of the shares of the IMP company to be sold, without actually holding the share (short sale).

4. Conclusions

Applying the Markowitz model on a portfolio of three titles in order to identify the efficient portfolio led us to an illegitimate portfolio.The results of the first three lines of the X matrix represents the percentage that it will be invested in the shares of those three companies, respectively IMP, COTR and COMI in order to obtain the efficient portfolio. It can be noticed that on the line which correspond to the IMP company there are negative weights, which means that the portfolio is illegitimate.

The research regarding the structure of the optimal portfolio, for each scenario, led us to the conclusion that it is necessary to sell a part of the titles of the IMP company in absence, procedure known as short sale, operation which is not regulated on the romanian capital market. This sales technique enables the companies to increase their investment budget, selling, even if they don't hold, securities that can be redeemed at their rate of expected return. Concerning the other two companies (COTR and COMI), based on the analysis of the 5 scenarios of portfolios chosen by completing the column vector K with random values of the expected return (E*p), it results the weights recommended to be invested, in order to obtain an efficient portfolio.

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