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MOLDOVA GDP FORECASTING USING BAYESIAN MULTIVARIATE MODELS

Ion PÂRȚACHI[1], Simion MIJA[2]

[1]Academy of Economic Studies of Moldova, 0000-0002-8042-983X
[2]Academy of Economic Studies of Moldova, 0009-0000-0814-5982

Abstract:
Building a multivariate GDP forecasting model based on relevant macroeconomic indicators selected through a proper selection process. This paper assesses whether alternative specifications of the Bayesian model can provide higher forecast accuracy compared to a standard VECM (Vector Error Correction Model). To achieve this, a Bayesian VAR (Vector Autoregressive) model is estimated using the Litterman precedent (1979). Compare the result based on the Bayesian VAR (Vector Autoregressive) model with the DFM (Dynamic Factor Model). The out-of-sample forecast performance of the models is then evaluated over a 5-year period (20 quarters), where model efficiency for a long forecast period is ascertained.

Keywords: *GDP Forecast, Econometric Models, Bayesian VAR*

JEL classification: *C10, C22, C38, C52*

1. Introduction

As a result of the most accurate assessment of the current situation regarding economic activity and its pressures on prices, interest in early assessments of the evolution of economic activity has increased. The specialised literature includes more and more references regarding the forecast of the real GDP growth rate in the short-term and its successive revisions.

Many developed forecasts often do not provide a clear and explicit presentation of the methodology used for forecasting and evaluating the current economic activity situation. Therefore, it is difficult to replicate and intuitively understand the forecasts. It is worth noting that forecasts, whether explicitly or implicitly, rely on expert judgment, and these experts may try to minimise forecasting errors through their biases. However, this approach carries two serious disadvantages. The first disadvantage is that expert biases turn the forecasting process into a black box, making it clear only to those who developed the forecast. The second disadvantage is that forecasting relies on human judgment and represents a subjective exercise rather than a quantitative analysis. In this regard, the experts creating the forecast may consume news and, as a result, be influenced by certain aspects that may not accurately reflect the current economic situation. However, at the same time, expert forecasts can influence the formation of expectations, and-in cases where they cannot be objectively quantified, they should be regarded only as a partial description of the economic situation. To avoid such issues, transparency in the methodological process is essential.

In this paper, the primary focus is on economic activity. The most relevant indicator for economic activity is Gross Domestic Product (GDP). It is reported quarterly and is published with a delay of approximately 70 days in the case of the Republic of Moldova. In this context, several methods are proposed to forecast the GDP growth rate based on the data available at monthly and quarterly frequencies before the National Bureau of Statistics releases the official data. The forecasting methods suggested are developed using: *(1) Dynamic Factor Model (DFM)*; *(2) Vector Error Correction Model (VECM);* and *(3) Bayesian Vector Autoregressive (VAR) model*.

[1] *ipartachi@ase.md*
[2] *mija.simion@ase.md*

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The algorithms of the proposed models for forecasting are designed to adapt to new information based on the decisions of the experts who develop them. Simultaneously, they avoid the significant inconveniences mentioned before. These forecasting methods are transparent, easy to interpret, straightforward to reproduce, and simple to update.

1.1 DFM model

The Dynamic Factor Model (DFM) is based on macro-econometrics, according to Stock and Watson (2001), and is consistent with the idea that the real GDP growth rate dynamics can be decomposed into two components:

- the first component pertains to the dynamics of the common factor.
- the second component pertains to the dynamics of the idiosyncratic factor.

The common factor represents the shared trends and patterns in economic data that affect GDP growth across different sectors, while the idiosyncratic factor captures the sector-specific or unique movements in economic data that are not shared across all sectors.

In recent years, specialised literature has extensively used two types of dynamic factor models. A model developed by Angelini (2008), which is closely related to the dynamic factor model, is based on a wide range of indicators and has been applied in the case of the Eurozone economy. The same type of model has been applied by Camacho and Sancho (2003) to the economy of Spain.

The model developed by Camacho and Sancho relies on a much smaller set of indicators and makes estimates strictly based on the common factor model. This alternative approach has been further developed by Camacho and Peres-Quiros (2018, 2013, 2008) for the Spanish economy and by Burriel and Garcia Belmonte (2013) for Eurozone data.

A common dynamic factor model, based on a relatively small number of indicators, aims to describe the current economic activity and its impact on price pressures and estimate the current GDP growth rate in the Republic of Moldova.

According to Mariano and Murasawa (2003), the quarterly average can be approximated by the monthly geometric average; therefore, the quarterly GDP growth rate can be expressed as the geometric average of the monthly growth rates, which are unobservable data:

$$y_t = \frac{2}{3}x_t + \frac{1}{3}x_{t-1} + x_{t-2} + \frac{1}{3}x_{t-3} + \frac{1}{3}x_{t-4}$$

The model is designed to provide timely and relevant data regarding the real GDP growth rate before the official data is released by the National Bureau of Statistics. First and foremost, the model incorporates indicators with higher volatility, encompassing the extremes of the economy to capture as much available information as possible. Secondly, the model includes data series with mixed frequencies to determine the evolution of GDP, which is reported quarterly, based on operational information, i.e., monthly indicators. Thirdly, the model is kept simple and can be automatically updated to consider potential economic instabilities. If the predictive power of one of the variables decreases for a period of time, that variable will have a lower weight in determining the aggregate factor.

1.2 VECM Model

Building on the research of Granger (1981) and Engle and Granger (1987), Vector Error Correction Models (VECMs) are essentially VARs with specific constraints. These models include a set of variables in both their differentiated forms and levels. The differences between the variables capture short-term relationships, while the linear combination of the variable levels, known as the cointegrating vector (or vectors, as multiple linear combinations may be included), represents the long-term dynamics of the variables. Mathematically, a typical VECM model can be expressed in matrix notation as follows:

$$\Delta y_t = m + \sum_{i=1}^{p-1} B_i \Delta y_{t-i} + A\Delta y_{t-1} + \varepsilon_t$$

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Where: m is a vector containing the constants of the equation system; Bi is a matrix that holds the coefficients describing the short-term impact of the variables' lag "i"; A is a matrix that contains the coefficients describing the long-term relationships between the variables. Additionally, the model can be extended to include exogenous variables that influence the system. VECMs are valuable for modeling non-stationary time series while preserving their long-term behavior. However, like unrestricted VARs, they face the "curse of dimensionality," meaning that adding more variables significantly increases the number of coefficients that need to be estimated.

1.3 BVAR Model

In simpler terms, Bayesian Vector Autoregressions (BVARs) offer an alternative to Ordinary Least Squares (OLS) VAR models. They were initially introduced by researchers like Sims (1980), Doan, Sims, and Litterman (1984) to enhance the forecasting performance of econometric models available at that time.

Under the Bayesian approach in econometrics, we don't estimate model coefficients in an attempt to find their true values. Instead, we treat these coefficients as the most likely values in a distribution. This distribution can be assumed to be normal in many cases. Moreover, using the Bayes theorem, a researcher can incorporate prior knowledge about the data as constraints, known as priors. When applying this technique, the estimated coefficients essentially become a combination of the imposed priors and a standard OLS estimate, weighted by the data:

$$\hat{b} = [V^{-1} + \mathrm{E}_e^{-1} * (X'X)]^{-1}(V^{-1}\bar{b} + \mathrm{E}_e^{-1} * X'Y)$$

Where: V^{-1} is the variance of (b); E_e^{-1} is the variance-covariance matrix of the residuals; X, Y are the variables included in the model.

To estimate the error variance-covariance matrix necessary for coefficient estimation in a Bayesian Vector Autoregression (BVAR), several methods can be used: (1) Fitting an AR (1) model: This approach involves fitting an AR (1) model to each variable individually to obtain the error variances. Each variable is assumed to follow an autoregressive process to capture the temporal dependencies; (2) Estimating an AR (1) and a VAR: In this method, an AR (1) model is estimated for each variable, and a Vector Autoregression (VAR) model is used to obtain the diagonal elements of the variance-covariance matrix. This approach considers both the autoregressive structure and the contemporaneous relationships between variables; (3) Estimating all variances and covariances with a full VAR: While less common, some models estimate all variances and covariances by specifying a full VAR, allowing for the complete modeling of relationships among all variables. However, this can lead to a singular matrix and is not widely used.

By imposing priors, the parameter space that OLS would typically need to explore for coefficient estimation is restricted. This results in a more parsimonious model that often outperforms traditional OLS models in terms of forecasting accuracy. It's important to note that the priors applied to coefficient estimation don't necessarily need to align with specific economic theories. They should primarily be consistent with the time-series properties of the variables included in the BVAR, helping to enhance the model's performance.

In this paper, our focus is on the Minnesota prior, a prior formulation introduced by Litterman (Litterman, 1979) and researchers at the University of Minnesota. The Minnesota prior is a Bayesian Vector Autoregression (BVAR) prior that shapes the coefficients in a way that makes the variables in the model appear as if they follow random walks. This prior is often used to incorporate additional information or structure into the BVAR model, allowing it to capture certain temporal dependencies and characteristics of the data, such as the tendency for variables to evolve as random walks. This approach has been employed by various researchers to improve the modeling of economic and financial time series.

In the context of Bayesian estimation in a Vector Autoregression (VAR) model, a set of parameters, often referred to as hyper-parameters, are defined by the researcher to influence the estimation of coefficients. Specifically, 1,1,2,3 are the hyper-parameters that serve as the prior for the

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mode (or central value) of the coefficients (b) of the first-order lags of variables in their own equations. This means it represents the researcher's prior belief about where the coefficients are likely to be centered.

When dealing with a variable that has a unit root, a researcher would typically select a value for 1, in the Bayesian Vector Autoregression model, that is very close to 1. This choice signifies the researcher's prior belief that the coefficients for the first-order lags of this variable should be strongly influenced by their past values, reflecting the inherent persistence of variables with unit roots. In simpler terms, setting 1 near 1 helps the model capture the long-lasting effects and relationships associated with variables that exhibit unit root behavior, enhancing the accuracy of the model's predictions for such variables.

If the variable is considered stationary (meaning it doesn't exhibit a unit root), the researcher would typically choose a value close to zero for 1. In such cases, the hyper-parameters 1,2 and ,3 come into play to shape the V-1 matrix, with 1 specifically indicating the strength or stringency of the constraints applied. A value of 1 close to zero means the more restrictions are in the estimation of the coefficients. The 2 varies between 0 and 1 and determines the cross-variable effect. And, finally, 3 responds for the importance of the own lags of a variable, excluding the first lag.

2. Data, empirical analysis, and estimation

The data samples used are from 2000Q1 to 2023Q2 and are seasonally adjusted through X-12 ARIMA (for monthly frequency data) and TRAMO/SEATS (for quarterly data) and sourced from the National Bureau of Statistics and the National Bank of Moldova.

DFM Model. For the economy of the Republic of Moldova, the DFM model was used with some adjustments by the author, namely: real GDP, annual growth (Y); on the supply side, the industrial production volume index by types of activities, annual growth (IPI); on the demand side, the turnover in retail trade, except for motor vehicles and motorcycles (Trade); on the side of incomes and aspects of the labour force in the absence of high-frequency data on the payroll and the number of employees were selected: the transfers of money from abroad (Transfer), the money supply (M2), and global agricultural production (Agro). The descriptive statistics of the indicators is represented in table 1.

Table 1: Descriptive statistics of data, DFM model

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Real GDP (annual, %)	**3.81**	**5.02**	**18.65**	**-13.6**	**5.78**
IPI (annual, %)	2.97	4.0	50.87	-30.1	
Trade (annual, %)	7.26	7.25	27.29	-9.35	
Transfer (annual, %)	16.37	13.08	80.8	-41.8	
M2 (annual)	6.09	6.62	18.31	-18.31	
Agro (annual, %)	3.24	2.10	125.1	-44.09	

Source: Prepared by the author, based on NBM and NBS data

Let x_t be the real GDP growth rate with monthly frequency and let z_t be the vector of macroeconomic indicators represented in annual growth rates of size k, where k is the number of macroeconomic indicators used in the model.

The model can be written:

$$\frac{x_t}{z_t} = \beta f_t + (\frac{u_{yt}}{u_{zt}})$$

Where: $u_{zt} = (u_{1t}, u_{2t}, \dots, u_{kt})$, and the parameters $(k+1)$ from β matrix are known as factors tasked with capturing the correlation between the unobserved common factor and the variables in the model, $\varphi_y(L)u_{yt} = \varepsilon_{yt}$, $\varphi_f(L)u_{ft} = \varepsilon_{ft}$, $\varphi_i(L)u_{it} = \varepsilon_{it}, i = 1, \dots, k$.
Where: $\varphi_y(L)$, $\varphi_f(L)$, $\varphi_i(L)$ are lags of order p, q and respectively r. Additionally, the errors within the equations are assumed to be independent, and normally distributed, with a zero mean and a diagonal covariance matrix. The form of representation of data on quarterly GDP growth and annual increases of indicators according to the model within the vector is: $Y_t = (y_t, \mathrm{z}_t')'$, and for their idiosyncratic component in the vector $u_t = (u_{yt}, \mathrm{u}_{zt}')'$.

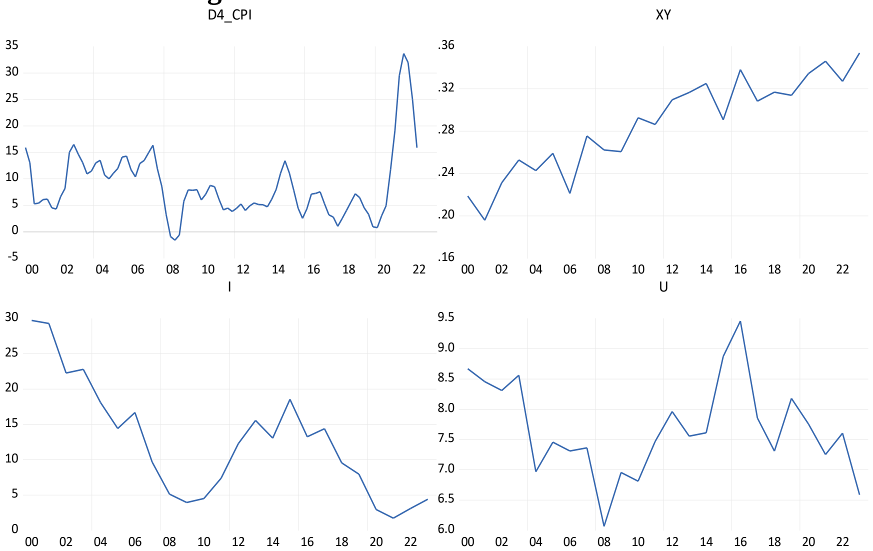
The form of the measurement equation, which describes the relationship between the state of the process and the measurements taken, is:

$$Y_t = HS_t + w_t$$
$$S_t = FS_{t-1} + v_t$$

Where: S_t is estimated based on a discrete-time controlled process. The random variable vectors w_t and v_t represent measurement noise and process noise, respectively. The measurement noise w_t and the process noise v_t are assumed to be independent, white noise, and normally distributed: $w_t = iN(0,R)$; $v_t = iN(0,Q)$, where R and Q are the process noise covariance and measurement noise covariance matrices, respectively, assumed to be constant.

VECM Model. VECM incorporates data for various economic indicators. These indicators include *real GDP* (Y); *unemployment rate* (u); *CPI inflation* (p); *91 days Treasury Bill* (i); and *exports as a percentage of GDP* (xY).

Figure 1: Macroeconomic variables in %



Source: Prepared by the author, based on NBM and NBS data

Each variable was tested by Augmented Dickey-Fuller (ADF) unit root test. However, the data was adjusted by taking the log differences and the variables became stationary, meaning they no longer showed a unit root behavior and were more suitable for modeling in a Vector Error Correction Model (VECM).

Table 2: Augmented Dickey-Fuller Unit Root Test

	ADFTest Statistic	Critical value5%	Critical value10%	Order	Remark
Real GDP	-9.57	-3.46	-3.15	I(1)	Stationarity
Unemp. rate	-13.28	-3.46	-3.16	I(1)	Stationarity
CPI inflation	-4.57	-3.45	-3.15	I(1)	Stationarity

91 days TBills	-7.53	-3.45	-3.15	I(1)	Stationarity
Export (% of GDP, diff)	-9.32	-3.45	-3.25	I(1)	Stationarity

Source: Prepared by the author, based on EViews 11

Table 3: VECM estimation output

Cointegrating Eq:	CointEq1	CointEq2			
L_Y(-1)	1.000000	0.000000			
L_CPI(-1)	0.000000	1.000000			
I(-1)	-5.087100	-7.434428			
	(1.01582)	(1.44393)			
	[-5.00786]	[-5.14874]			
U(-1)	-11.67878	-13.67600			
	(4.85254)	(6.89760)			
	[-2.40674]	[-1.98272]			
XY(-1)	-530.7245	-912.1819			
	(103.659)	(147.345)			
	[-5.11990]	[-6.19077]			
C	-717.8466	-233.5582			
Variables:	D(L_Y)	D(L_CPI)	D(I)	D(U)	D(XY)
CointEq1	-0.152857	0.035058	0.059330	0.029479	-0.000406
	(0.04953)	(0.02314)	(0.04326)	(0.01949)	(0.00048)
	[-3.08586]	[1.51485]	[1.37144]	[1.51241]	[-0.83849]
CointEq2	0.111655	-0.022587	-0.011070	-0.019713	0.000366
	(0.03605)	(0.01684)	(0.03148)	(0.01418)	(0.00035)
	[3.09754]	[-1.34120]	[-0.35165]	[-1.38979]	[1.03823]
D(L_Y(-1))	0.018272	0.006455	-0.180781	-0.080567	0.001826
	(0.10554)	(0.04931)	(0.09217)	(0.04153)	(0.00103)
	[0.17312]	[0.13091]	[-1.96128]	[-1.93998]	[1.76890]
D(L_CPI(-1))	-0.003241	0.659318	0.878271	-0.010611	0.000488
	(0.20801)	(0.09718)	(0.18167)	(0.08185)	(0.00203)
	[-0.01558]	[6.78431]	[4.83453]	[-0.12964]	[0.24003]
D(I(-1))	-0.052881	0.015790	0.128022	-0.022487	0.002519
	(0.10896)	(0.05091)	(0.09516)	(0.04288)	(0.00107)
	[-0.48531]	[0.31016]	[1.34529]	[-0.52447]	[2.36287]
D(U(-1))	-0.049036	-0.149632	0.373288	-0.308823	-0.001993
	(0.27068)	(0.12646)	(0.23640)	(0.10651)	(0.00265)
	[-0.18116]	[-1.18322]	[1.57907]	[-2.89948]	[-0.75261]
D(XY(-1))	-0.285662	4.782555	9.289040	-2.867599	-0.052251
	(11.8177)	(5.52128)	(10.3211)	(4.65020)	(0.11560)
	[-0.02417]	[0.86620]	[0.90001]	[-0.61666]	[-0.45199]
C	0.896475	0.668045	-1.756088	0.043218	0.002042
	(0.52708)	(0.24626)	(0.46033)	(0.20740)	(0.00516)
	[1.70082]	[2.71282]	[-3.81484]	[0.20838]	[0.39605]
R-squared	0.406980	0.504011	0.406161	0.366514	0.346209

Source: Prepared by the author, based on EViews 11

BVAR Model. Excluding μ_1, which is set to zero because the model is estimated using logarithmic differences, and these differences make the variables stationary, the other hyper-parameters required for obtaining coefficient estimates through the Minnesota-Litterman prior are chosen through a process of trial.

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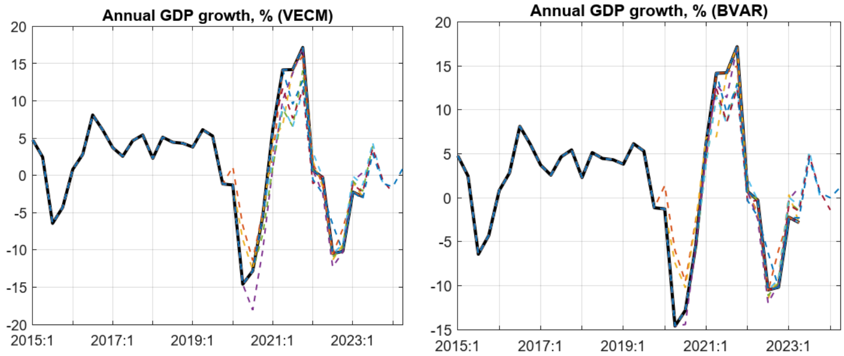
The combination of hyper-parameters that results in a model with independent and identically distributed (iid.) residuals is as follows: $\lambda_1 = \mathbf{6}, \lambda_2 = 0.2, \lambda_3 = \mathbf{0.1}$ **(**

For each variable, we apply an AR(1) model to determine the variances of the residuals. The covariances, which specifically pertain to the diagonal elements of the variance-covariance matrix, are extracted from the equivalent matrix of the corresponding ordinary least squares (OLS) vector autoregression (VAR) model. Additionally, as suggested by the length criteria, the lag length of the model is set to 2.

3. Results and forecast evaluation

To evaluate the forecasting performance, a forecast exercise was produced, where the two models (VECM and BVAR) are estimated up to time t and perform a forecast of the quarters t+1 to t+4. They are then estimated up to t+1 and perform a forecast of the quarters t+2 to t+5, and so on. Overall, 20 recursive estimations are performed from 2019Q3 up to 2023Q2, with the last forecast being that of 2023Q3-2024Q2.

Figure 2: In-sample forecast based on VECM and BVAR



Source: Prepared by the author, based on NBS data (MATLAB 2022)

After generating the forecasts, their quality is assessed using three common evaluation metrics: Root Mean Squared Error (RMSE).

4. Conclusions

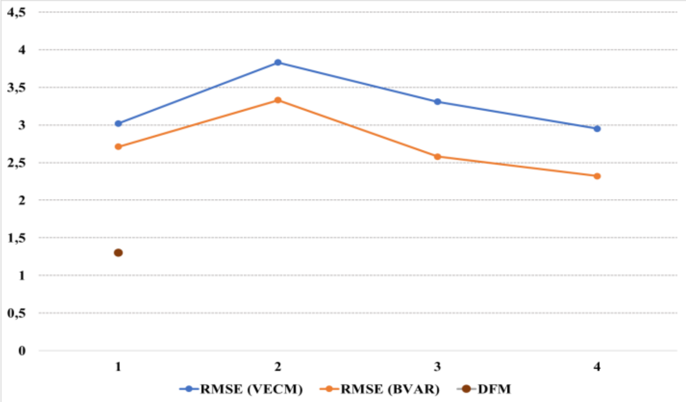
Based on the results, the Bayesian models deliver better results compared to the standard VECM model.

More specifically, the BVAR with the Minnesota prior provides a more accurate forecast over the entire period, compared with the standard VECM model.

The largest measurement errors are recorded in the 2nd quarter of the forecast, both for the VECM model and in the case of using the BVAR model.

A special case is the DFM model. It evaluated the deviations of the forecast values from the actual data only for the first forecast period, for 10 cases, and the RMSE result was 1.3.

Figure 3: RMSE based on VECM, BVAR, and DFM (first period)



Source: Prepared by the author

Important aspects are determined by the fact that, from the beginning, it is crucial to appreciate what type of forecast is desired. If a forecast is desired for a short period, for example, 1 quarter, then it would be optimal to use a DFM model, based on information based on data with a higher frequency.

If it is desired to make a forecast over a longer period, for example, a year, as in the case of this work, then it is recommended to use a BVAR model to the detriment of a VECM model. One more positive aspect is that this model can also be applied to conduct impulse response analysis, enabling an exploration of how exogenous shocks would impact the economy in varying ways at different time instances.

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