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OPTIONS EVALUATION USING MONTE CARLO SIMULATION

Vasile BRĂTIAN [1]

Lucian Blaga University of Sibiu, Romania

Abstract

The present paper evaluates derivative products as options, using Monte Carlo simulation for the support-asset. The Monte Carlo method is one of the most valuable and used methods in modern finance and with great applicability in the pricing of options. The support-asset used in our developments is the shares of Banca Transilvania SA. The Monte Carlo simulation is used by us to create scenarios on the random evolution of the support-asset, and the price of the option is determined using the Feynman-Kac theorem. We also consider that the price of the support-asset follows a stochastic process with a lognormal distribution.

Keywords: Monte Carlo simulation, Feynman Kac theorem, options price, brownian motion.

JEL classification: C02, C15, G13

1. Introduction

Monte Carlo Simulation is a tool that is widely used in quantitative finance for the evaluation of derivative products of the nature of options, being a highly used method. Using it to create scenarios on the evolution of the support asset and the Feynman-Kac theorem for determining solutions for CALL and PUT, they can be a very useful way for practitioners to evaluate options. What we are proposing in this paper is to describe theoretically and practically the way in which this goal can be achieved.

Partial differential equation of second order, parabolic, used in the evaluation of options, linked to the heat equation in mechanics, is proposed for

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[1] *Assoc. Prof. PhD,Faculty of Economics, Department of Finance and Accounting, Lucian Blaga University Sibiu, Sibiu, Romania, e-mail: vasile.bratian@ulbsibiu.ro.*

the scientific debate by Fischer Black and Myron Scholes (1973) and later developed by Robert Merton (1973). This equation is one of the equations that have changed the world in the view of mathematician Ian Stewart, along with other 17 equations considered relevant (Einstein equation, Maxwell equations, Boltzmann equation, Schrödinger equation, Navier-Stokes equation, wave equation, normal distribution, etc.). *”The Black-Scholes equation has changed the world, creating a million-billion-dollar industry, but its generalizations, used in an unintelligible way, by a small group of bankers, have changed the world again by contributing to financial collapse of one million billion.”* (Stewart, 2013, p. 272).

Our paper refers to support assets, shares quoted on the Romanian capital market (our case study is built on this market and we consider the option to be of European type). Romania has an option market since 1998, but is currently in the process of institutional reorganization due to a merger. In 1994, is created in Romania, in Sibiu, the Sibiu Financial and Commodities Exchange, which later becomes the Sibiu Stock Exchange (SIBEX), where the first options on futures are launched in 1998, and in 2011 contracts are launched on options on the euro/dollar exchange rate. The Sibiu Stock Exchange becomes a member of the Swiss Futures and Options Association and in 2017 it ceases its activity through the absorption merger by the Bucharest Stock Exchange.

2. Literature review

The area of quantitative finance in which we find the topic we are approached is quite wide, and the researchers' concerns on this issue are numerous. Among the first to explore the choice of options using the Monte Carlo simulation are: Phelim Boyle, who studied the price of European options (Boyle, 1977); Mark Broadie and Paul Glasserman, who studied the price of Asian options (Broadie, Glasserman, 1996); Francis Langstaff and Eduardo Schwartz, who studied the price of American options (Langstaff, Schwartz, 2001).

Along with those mentioned above, we have several landmarks, reminding: John Charnes (2000), Michael Giles (2007), Long Yun (2010), Russel Caflisch and Suneal Chaudary (2004), Claus Jespersen (2015), Bingqian Lu (2011), Ajay Jasra and Pierre del Moral (2010), Bolia and Juneja (2005), Ivan Popchev and Nadya Velinova (2003).

3. Methodology

An important premise in the evaluation of options is that the price of the support asset *(S)* has a random evolution given by the expression:

$$dS = \mu S dt + \sigma S dB \qquad (1)$$

where: S = the price of the support asset; μ = drift; σ = volatility of the support asset; B = brownian; t = time.

According to Black-Scholes theory, the value of the option is the present value of the expected maturity payment for a random neutral risk evolution of the support asset. The random neutral risk outcome for *S* can be obtained with the *Girsanov theorem* (Bratian (coordinator), Bucur, Opreana, 2016, p. 427):

Theorem: Let the probability field (Ω, F, P), *dB* - the Brownian motion, F_t - a filter generated by B_t , and the stochastic process θ_t.
We define the following probability measure: $Q(F) = \int_F L_T dP$, where: $L_T = \left\{-\int_0^t \theta_t dB_t - \frac{1}{2}\theta_t^2 dt\right\}, t \in [0, T]$. Then the process $dB^* = dB + \theta dt$ is a brownian motion relative to the measure *Q*.

Note: P is the real probability and Q is the neutral-risk probability.

Applying Girsanov's theorem to $\theta = \frac{\mu - r}{\sigma}$ we have:

$$dB^* = dB + \frac{\mu - r}{\sigma} dt \qquad (2)$$

where: r = risc-free interest rate.

From (1) and (2) we can write successively the following:

$$dS = \mu S dt + \sigma S \left(dB^* - \frac{\mu - r}{\sigma} dt\right)$$

$$dS = \mu S dt + \sigma S dB^* - \sigma S \frac{\mu - r}{\sigma} dt$$

$$dS = (\mu - \mu + r)Sdt + \sigma SdB^*$$

$$dS = rSdt + \sigma SdB^* \tag{3}$$

Therefore, according to Girsanov's theorem, σ is independent of μ and the volatility of the support asset price is the same in the real and the risk-free market.

It is very important to remember that the probability change with the Girsanov theorem allows refocusing brownian motion. The transformation of Girsanov changes the instantaneous drift of the process, but does not change the diffusion coefficient (Negrea, 2006, p. 77).

Next, for the lognormal random evolution, the above neutral risk stochastic differential equation can be written as follows:

$$d(lnS) = \left(r - \frac{1}{2}\sigma^2\right)dt + \sigma Z\sqrt{dt} \tag{4}$$

Expression (4) can be integrated and the following equation of motion is obtained:

$$lnS(t) - \ln(0) = \left(r - \frac{1}{2}\sigma^2\right)t + \sigma(B(t) - B(0)) \tag{5}$$

As a result, the solution for a time step is:

$$S(t) + \Delta t = S(t)e^{\left[\left(r - \frac{1}{2}\sigma^2\right)\Delta t + \sigma Z\sqrt{\Delta t}\right]} \tag{6}$$

where: B(t) is a Gaussian process; B(t) – B(0) = $Z\sqrt{t}$; $Z \sim N(0{,}1)$.

That being said, now, if we note with *V(S,t)* the value of the option, the Black-Scholes equation tells us:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + r\left(S\frac{\partial V}{\partial S} - V\right) = 0 \tag{7}$$

Referring to physics, the meaning of the terms in the Black-Scholes equation is the following (Wilmott, 2007, p. 159):

- expression $\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}$ means that we are dealing with a diffusion in a homogeneous environment;
- expression $rS\frac{\partial V}{\partial S}$ is the term of convection (in a physical system, convective is due to a wind breeze that spreads the smoke in a certain direction);
- expression $-rV$ is the term of reaction (by balancing this term and derivation with time, we will obtain a model for the collapse of a radioactive body).

The above equation does not specify the option category, CALL or PUT, which is being evaluated and the exercise price or maturity. What we know is that the value of the option is a function of the support asset value at maturity. This means that we have to write a function $V(S_T, T)$ representing profit or loss at maturity. So, if we have a CALL option, then we know that (Wilmott, 2002, p. 97):

$$V(S_T,T)=\max(S_T-E,0)=Payoff(S), \qquad (8)$$

and for PUT we have:

$$V(S_T,T)=\max(E-S_T,0)=Payoff(S) \qquad (9)$$

where: E = the exercise price; T = the time of maturity.

Next, the *Feynman-Kac* theorem tells us that if we have a $V=V(S,t)$ function differentiable by S and t, given by the following successive phrases (Bratian (coordinator), Bucur, Opreana):

$$dV=\frac{\partial V}{dt}+r(S,t)\frac{\partial V}{\partial S}+\frac{1}{2}\sigma(S,t)^2\frac{\partial^2 V}{\partial S^2}-r(S,t)V(S,t)=0$$

$$dV=\frac{\partial V}{\partial t}+rS\frac{\partial V}{\partial S}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}-rV=0$$

$$dV = \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + r\left(S\frac{\partial V}{\partial S} - V\right) = 0,$$

and put the condition at the limit $V(S_T, T)$, and $r(S,t)$ is constant, the value of a derivative with payoff has as the only solution:

$$V(S,t) = e^{-r(T-t)}E^Q[(V(S_T,T)|F_t)] \tag{10}$$

Therefore, on the basis of equations (8), (9) and (10) we can write the solutions for CALL and PUT as follows:

- for CALL we have:

$$C = V(S,t) = e^{-r(T-t)}E^Q\left[\max\left((S_T - E, 0)\middle|F_t\right)\right] \tag{11}$$

- for PUT we have:

$$P\ = V(S,t) = e^{-r(T-t)}E^Q\left[\max\left((E - S_T, 0)\middle|F_t\right)\right] \tag{12}$$

The above Feynman-Kac representation theorem, actually provides the probabilistic solution of the Black-Scholes partial derivative equation (Negrea, 2006, p. 136).

4. Evaluation of options on suport-asset the shares of Banca Transilvania SA using the Monte Carlo simulation

In the following, we will empirically address the issue of evaluating the options on non-dividend sharesm, using the above methodology. In this respect, we consider that Transilvania Bank's shares as support asset are best suited to our actions, as this bank has a specific clientele that has proven over time that it is not directly interested in the dividend.

For the calculation of the volatility of the support asset we use data of the share price of Banca Transilvania on a year of trading (01 / July / 2016 - 04 / July / 2017), and the risk-free interest rate is 0.94% (http://www.bnro.ro/Titluri-de-stat---rate-de-referinta-(fixing)-6332.aspx).

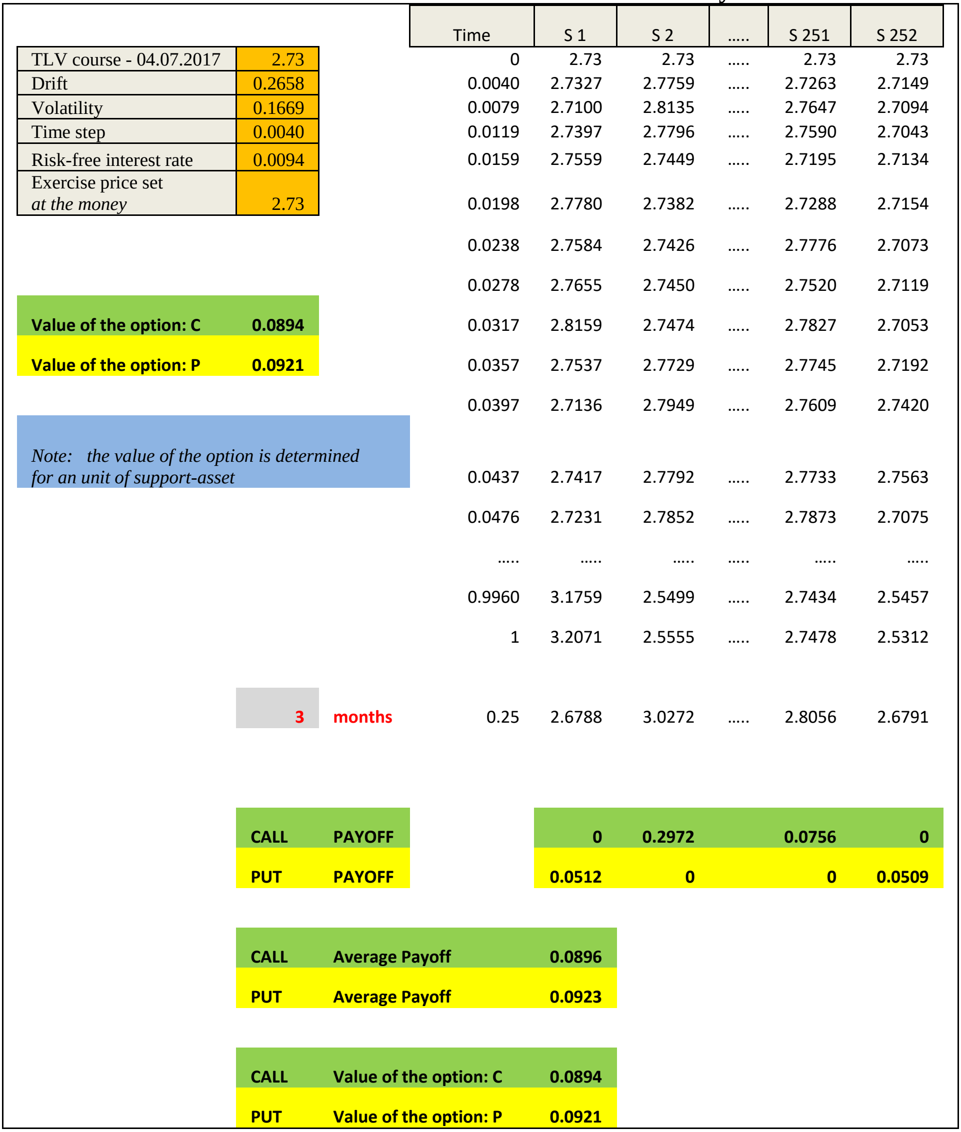
Simulation using the Monte Carlo method of the support asset price is realized starting with 04 / July / 2017 using the equation (6) of the methodology and we created 252 daily scenarios.

The payoff and value of CALL and PUT options for a unit of TLV share support asset are calculated for the following maturities: 3 months, 6 months, 9 months and 12 months.

Following calculations, the results for CALL and PUT for an asset-backed unit are as follows:

a) 3-month maturity (see Table 1)

Table 1: CALL/PUT with 3-month maturity



TLV course - 04.07.2017	2.73
Drift	0.2658
Volatility	0.1669
Time step	0.0040
Risk-free interest rate	0.0094
Exercise price set *at the money*	2.73

Value of the option: C **0.0894**

Value of the option: P **0.0921**

Note: the value of the option is determined for an unit of support-asset

Time	S 1	S 2	.....	S 251	S 252
0	2.73	2.73	.....	2.73	2.73
0.0040	2.7327	2.7759	.....	2.7263	2.7149
0.0079	2.7100	2.8135	.....	2.7647	2.7094
0.0119	2.7397	2.7796	.....	2.7590	2.7043
0.0159	2.7559	2.7449	.....	2.7195	2.7134
0.0198	2.7780	2.7382	.....	2.7288	2.7154
0.0238	2.7584	2.7426	.....	2.7776	2.7073
0.0278	2.7655	2.7450	.....	2.7520	2.7119
0.0317	2.8159	2.7474	.....	2.7827	2.7053
0.0357	2.7537	2.7729	.....	2.7745	2.7192
0.0397	2.7136	2.7949	.....	2.7609	2.7420
0.0437	2.7417	2.7792	.....	2.7733	2.7563
0.0476	2.7231	2.7852	.....	2.7873	2.7075
.....	.....	.....	.....	.....	.....
0.9960	3.1759	2.5499	.....	2.7434	2.5457
1	3.2071	2.5555	.....	2.7478	2.5312
0.25	2.6788	3.0272	.....	2.8056	2.6791

3 months

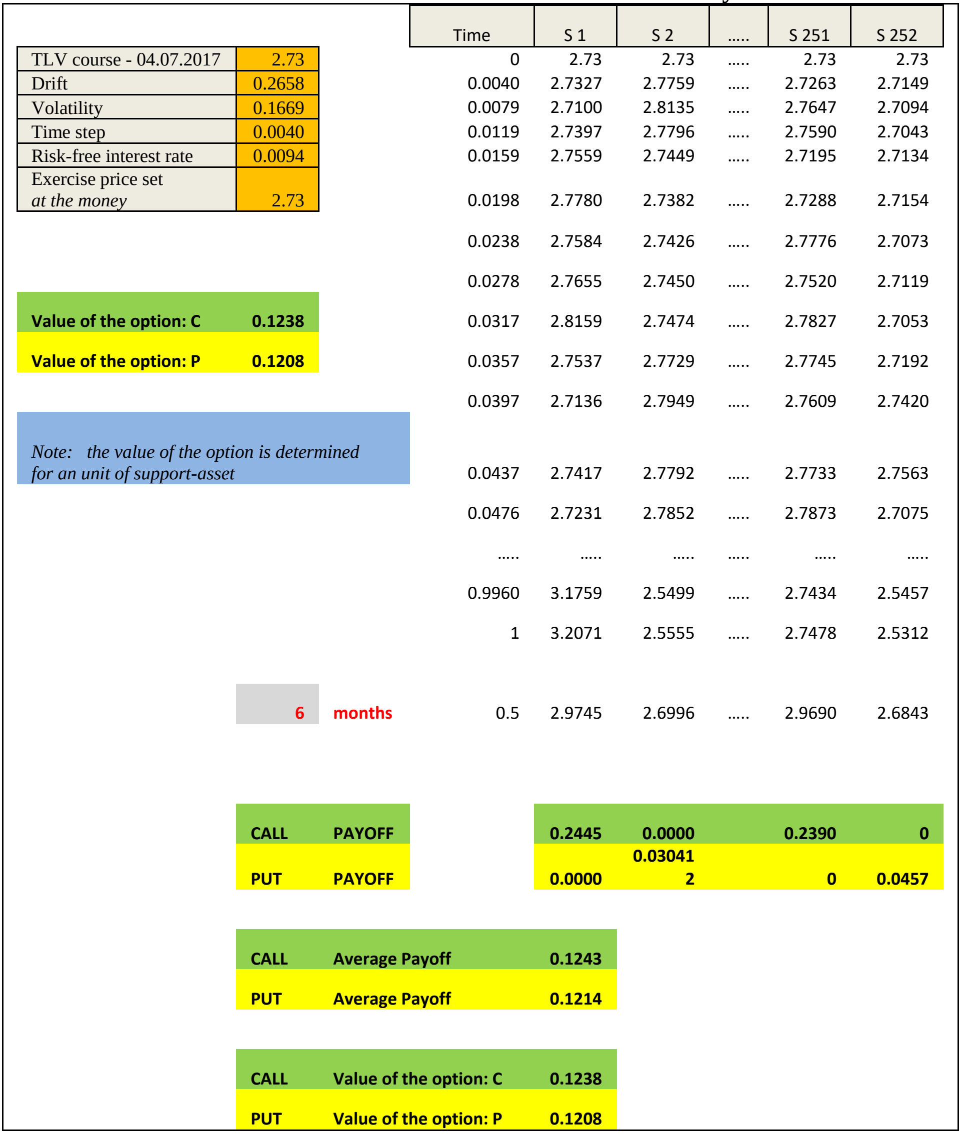
CALL	**PAYOFF**	**0**	**0.2972**	**0.0756**	**0**
PUT	**PAYOFF**	**0.0512**	**0**	**0**	**0.0509**

CALL	**Average Payoff**	**0.0896**
PUT	**Average Payoff**	**0.0923**

CALL	**Value of the option: C**	**0.0894**
PUT	**Value of the option: P**	**0.0921**

b) 6-month maturity (see Table 2)

Table 2: CALL/PUT with 6-month maturity



TLV course - 04.07.2017	2.73
Drift	0.2658
Volatility	0.1669
Time step	0.0040
Risk-free interest rate	0.0094
Exercise price set *at the money*	2.73

Value of the option: C	**0.1238**
Value of the option: P	**0.1208**

Note: the value of the option is determined for an unit of support-asset

Time	S 1	S 2	.....	S 251	S 252
0	2.73	2.73	.....	2.73	2.73
0.0040	2.7327	2.7759	.....	2.7263	2.7149
0.0079	2.7100	2.8135	.....	2.7647	2.7094
0.0119	2.7397	2.7796	.....	2.7590	2.7043
0.0159	2.7559	2.7449	.....	2.7195	2.7134
0.0198	2.7780	2.7382	.....	2.7288	2.7154
0.0238	2.7584	2.7426	.....	2.7776	2.7073
0.0278	2.7655	2.7450	.....	2.7520	2.7119
0.0317	2.8159	2.7474	.....	2.7827	2.7053
0.0357	2.7537	2.7729	.....	2.7745	2.7192
0.0397	2.7136	2.7949	.....	2.7609	2.7420
0.0437	2.7417	2.7792	.....	2.7733	2.7563
0.0476	2.7231	2.7852	.....	2.7873	2.7075
.....	.....	.....	.....	.....	.....
0.9960	3.1759	2.5499	.....	2.7434	2.5457
1	3.2071	2.5555	.....	2.7478	2.5312
6 months 0.5	2.9745	2.6996	.....	2.9690	2.6843
CALL PAYOFF	**0.2445**	**0.0000**		**0.2390**	**0**
PUT PAYOFF	**0.0000**	**0.030412**		**0**	**0.0457**

CALL	**Average Payoff**	**0.1243**
PUT	**Average Payoff**	**0.1214**

CALL	**Value of the option: C**	**0.1238**
PUT	**Value of the option: P**	**0.1208**

c) 9-month maturity (see Table 3)

Table 3: CALL/PUT with 9-month maturity

TLV course - 04.07.2017	2.73
Drift	0.2658
Volatility	0.1669
Time step	0.0040
Risk-free interest rate	0.0094
Exercise price set *at the money*	2.73

Value of the option: C	**0.1692**
Value of the option: P	**0.1357**

Note: the value of the option is determined for an unit of support-asset

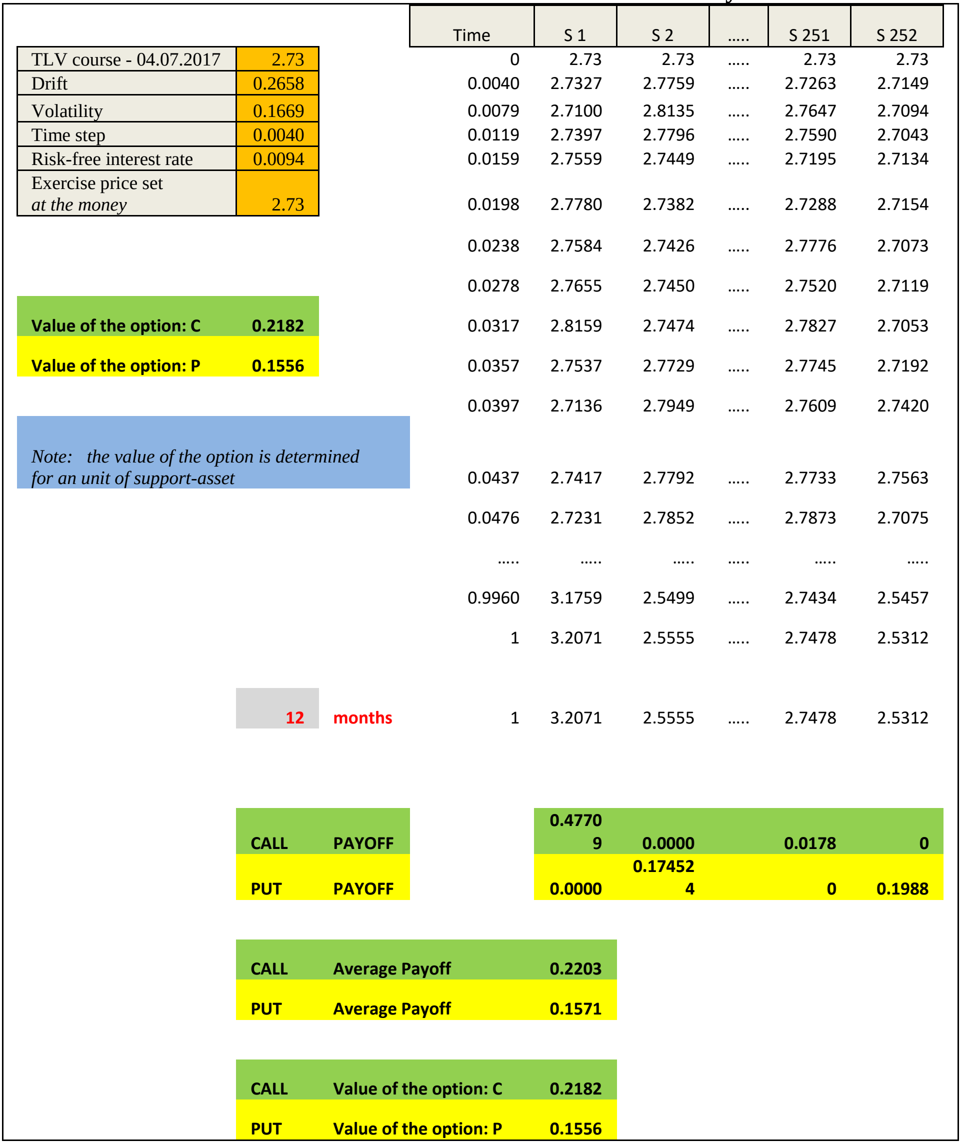
		Time	S 1	S 2	.....	S 251	S 252
		0	2.73	2.73	.....	2.73	2.73
		0.0040	2.7327	2.7759	.....	2.7263	2.7149
		0.0079	2.7100	2.8135	.....	2.7647	2.7094
		0.0119	2.7397	2.7796	.....	2.7590	2.7043
		0.0159	2.7559	2.7449	.....	2.7195	2.7134
		0.0198	2.7780	2.7382	.....	2.7288	2.7154
		0.0238	2.7584	2.7426	.....	2.7776	2.7073
		0.0278	2.7655	2.7450	.....	2.7520	2.7119
		0.0317	2.8159	2.7474	.....	2.7827	2.7053
		0.0357	2.7537	2.7729	.....	2.7745	2.7192
		0.0397	2.7136	2.7949	.....	2.7609	2.7420
		0.0437	2.7417	2.7792	.....	2.7733	2.7563
		0.0476	2.7231	2.7852	.....	2.7873	2.7075
		.....	.....	.....	.....	.....	.....
		0.9960	3.1759	2.5499	.....	2.7434	2.5457
		1	3.2071	2.5555	.....	2.7478	2.5312
9	months	0.75	3.2202	2.6659	.....	3.0059	2.7929
CALL	**PAYOFF**		**0.49016**	**0.0000**		**0.2759**	**0.062906**
PUT	**PAYOFF**		**0.0000**	**0.064127**		**0**	**0.0000**

CALL	**Average Payoff**	**0.1704**
PUT	**Average Payoff**	**0.1366**

CALL	**Value of the option: C**	**0.1692**
PUT	**Value of the option: P**	**0.1357**

d) 12-month maturity (see Table 4)

Table 4: CALL/PUT with 12-month maturity



TLV course - 04.07.2017	2.73
Drift	0.2658
Volatility	0.1669
Time step	0.0040
Risk-free interest rate	0.0094
Exercise price set *at the money*	2.73

Value of the option: C	**0.2182**
Value of the option: P	**0.1556**

Note: the value of the option is determined for an unit of support-asset

Time	S 1	S 2	.....	S 251	S 252
0	2.73	2.73	.....	2.73	2.73
0.0040	2.7327	2.7759	.....	2.7263	2.7149
0.0079	2.7100	2.8135	.....	2.7647	2.7094
0.0119	2.7397	2.7796	.....	2.7590	2.7043
0.0159	2.7559	2.7449	.....	2.7195	2.7134
0.0198	2.7780	2.7382	.....	2.7288	2.7154
0.0238	2.7584	2.7426	.....	2.7776	2.7073
0.0278	2.7655	2.7450	.....	2.7520	2.7119
0.0317	2.8159	2.7474	.....	2.7827	2.7053
0.0357	2.7537	2.7729	.....	2.7745	2.7192
0.0397	2.7136	2.7949	.....	2.7609	2.7420
0.0437	2.7417	2.7792	.....	2.7733	2.7563
0.0476	2.7231	2.7852	.....	2.7873	2.7075
.....	.....	.....	.....	.....	.....
0.9960	3.1759	2.5499	.....	2.7434	2.5457
1	3.2071	2.5555	.....	2.7478	2.5312

12	months	1	3.2071	2.5555	.....	2.7478	2.5312
CALL	**PAYOFF**		**0.47709**	**0.0000**		**0.0178**	**0**
PUT	**PAYOFF**		**0.0000**	**0.174524**		**0**	**0.1988**

CALL	**Average Payoff**	**0.2203**
PUT	**Average Payoff**	**0.1571**

CALL	**Value of the option: C**	**0.2182**
PUT	**Value of the option: P**	**0.1556**

Note: The above calculations were done in Excel.

5. Conclusions

Following the calculations, we can say the following:

a) The value of the CALL option for a quantity of a support-asset unit of the nature of the TLV share is approximately:
  - with a maturity of 3 months: 0,0894 monetary units (3,27% of the value of the support-asset on 04 / July / 2017);
  - with a maturity of 6 months: 0,1238 monetary units (4,53% of the value of the support-asset on 04 / July / 2017);
  - with a maturity of 9 months: 0,1692 monetary units (6,19% of the value of the support-asset on 04 / July / 2017);
  - with a maturity of 12 months: 0,2182 monetary units (7,99% of the value of the support-asset on 04 / July / 2017).

b) The value of the PUT option for a quantity of a support-asset unit of the nature of the TLV share is approximately:
  - with a maturity of 3 months: 0,0921 monetary units (3,37% of the value of the support-asset on 04 / July / 2017);
  - with a maturity of 6 months: 0,1208 monetary units (4,42% of the value of the support-asset on 04 / July / 2017);
  - with a maturity of 9 months: 0,1357 monetary units (4,97% of the value of the support-asset on 04 / July / 2017);
  - with a maturity of 12 months: 0,1556 monetary units (5,69% of the value of the support-asset on 04 / July / 2017).

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