THE SYNCHRONIZATION EFFECTS OF STOCK INDICES DYNAMICS IN THE MULTIFRACTAL ANALYSIS USING THE WAVELET TECHNOLOGY

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Abstract

The behavior of stock indices before and during the crisis is analyzed in order to identify the key characteristics of the pre-crisis state, to locate and to describe the crisis effects in time and scope using the multifractal analysis and the wavelet transformation.

The purpose of the article is the study of synchronization effects that arise at research of the dynamics of stock indexes by the methods of multifractal and coherent analysis using wavelet technology. The problem is to conduct multifractal analysis of stock indexes in Europe, USA, Asia and identify the synchronization effects of their behavior using wavelet coherence measures.

The proposed measure of synchronous behavior of stock indices allows to estimate the growth of the crisis and to predict them. There is a close relationship between its values and the power of crisis that took place in the appropriate time period.

The characteristics of major economic crises over the period 1997-2013 are presented to compare actual events and features of the dynamics of synchronization measures as warning of the crisis.

Keywords: *stock indices, fractal analysis, Hurst index, the wavelet transformation*

JEL classification: *C53, C45, G01,G17*

1. Introduction

Modern trends of economic and mathematical modeling are characterized by the transition from the linear to nonlinear models. Non-linear

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techniques aimed at the study of complex systems and are based on the principles of synergy. Quite important issue of the consideration is the analysis of the crisis in the financial systems, which dynamics is common. Financial instability is a threat of economic development. As a result, it is very important to research and develop the financial series of anti-crisis measures aimed at preventing and minimizing the effects of potential financial and economic crises.

The problem of analysis and modeling of behavior of the stock and currency markets, foreign exchange rates and market indexes reflected in [1-8]. The research of some financial time series of fractal analysis is conducted to determine their persistence at fixed time intervals and by moving the time window [1, 3, 6]. In articles [3-5, 7, 8] the approaches using continuous and discrete wavelet transforms, wavelet coherence [8, 9] were proposed.

Complex synergistic economies require a comprehensive approach to the study of their behavior based on modern methods of nonlinear dynamics.

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2. Research methods

One of the challenging areas of research of economic processes is the analysis of fractal and multifractal properties of financial time series. The popularity of fractal analysis is based on its ability to investigate signals in terms of spectral theory that are the white noise or Brownian motion [1, 2, 4]. The reason of of this popularity is the ability of fractal analysis to investigate signals in terms of covariance and spectral theory that are the white noise or Brownian motion.

For the mean square of the growth of self-similar process $x(t)$ is carried out $M\{|x(t+\delta\cdot t)-x(t)|^2\}\sim|\delta\cdot t|^{2H}$, where $0<H<1$ is Hurst constant, and the dependence of the power spectrum on the frequency is a power law in nature $S_{xx}(\omega)\sim\omega^{-(2H+1)},\omega\to 0$.

At present the method for the fluctuation analysis after excluding of scale-dependent trends (DFA) is widely used to estimate the singularity spectra of the time series [1, 4].

Let us consider the calculation of the singularity spectrum $F(\alpha)$, that can be defined as the fractal dimension of a set of points, in which neighborhood there is the Lipschitz - Hölder factor α for random realizations of the process $x(t)$, that is mean those points t, for which $|x(t+\delta)-x(t)| \sim |\delta|^{\alpha}$, $\delta \to 0$.

Suppose there is a finite sample of the time series $x(t),\ t=1,\dots,N$, s is a number of counts associated with a variable scale δ_s : $\delta_s = s\Delta t$. Let us divide the sample into small intervals that do not cross and have the length s in samples:

$$I_k^{(s)} = \{t : 1+(k-1)s \le t \le ks,\ \ k=1,\dots,[N/s]\} \tag{1}$$

$$y_k^{(s)}(t) = x((k-1)s+t),\ t=1,\dots,s \tag{2}$$

where $y_k^{(s)}(t)$ is a part of the time series $x(t)$, corresponding to the interval $I_k^{(s)}$. Let $p_k^{(s,m)}(t)$ is a polynom of degree m, that is fitted by the method of least squares to the signal $y_k^{(s)}(t)$. Consider the deviation from the local trend:

$$\Delta y_k^{(s,m)}(t) = y_k^{(s)}(t) - p_k^{(s,m)}(t),\ \ t=1,\dots,s \tag{3}$$

The standard approach consists of computing of the Gibbs statistical sum:

$$W(q,s) = \sum_{k=1}^{[N/s]} \left(\max_{1\le t\le s} \Delta y_k^{(s,m)}(t) - \min_{1\le t\le s} \Delta y_k^{(s,m)}(t) \right)^q \tag{4}$$

and determination of the mass $\tau(q)$ from the condition $W(q,s) \sim s^{\tau(q)}$ after that spectrum $F(\alpha)$ is calculated according to the formula:

$$F(\alpha) = \max\{\min_q(\alpha q - \tau(q)), 0\} \tag{5}$$

From the above formulas it follows that $\tau(q) = \rho(q) - 1 = qh(q) - 1$. So,

$$F(\alpha) = \max\{\min_q(q(\alpha - h(q)) + 1, 0\} \tag{6}$$

For monofractal process where $h(q) = H = const$, we have:

$$F(H) = 1 \text{ and } F(\alpha) = 0\ \forall \alpha \ne H \tag{7}$$

In the particular case, the position and width of the media spectrum $F(\alpha)$, i.e. the values $\alpha_{\min}$, $\alpha_{\max}$, $\Delta\alpha = \alpha_{\max} - \alpha_{\min}$ and α^* are the value that gives the maximum of the function $F(\alpha)$. The value α^* is called the generalized Hurst index. For the monofractal signal the value $\Delta\alpha$ should be zero, and $\alpha^* = H$. The value $F(\alpha^*)$ is the fractal dimension of points for which the neighborhood is performed by scaling ratio $M(\delta, q) \sim |\delta|^{\rho(q)}$.

If we estimate the spectrum $F(\alpha)$ in the sliding window, its evolution can provide the information about changes in the structure of chaotic fluctuation range.

Wavelet coherence measure is similar to the spectral measure, but is based on the decomposition of signals at orthogonal wavelet basis functions. According to this method it is estimated the scale-dependent measure of coherent behavior of time series $Z(t)$ in sliding time window of the length in N samples. Let the number of rows that are analyzed be $q \geq 3$, $Z(t) = \left(Z_1(t), ..., Z_q(t)\right)^T$, and τ is the position of the right end of the sliding time window of the length in N samples for time points that satisfy the inequality $\tau - N + 1 \leq t \leq \tau$. For each position of the time window (which is shifting right by one counting) the analysis is independent of the analysis of other windows.

Denote $Q^{(\tau,\beta)}$ the set $L_\beta\left(N\right) = 2^{-\beta} N$ of significant $q-$dimensional vectors $z = \left(z_1, z_2, ..., z_q\right)^T$ of wavelet coefficients of level of detail β, trapped in a time window τ. We divide the vector z into two parts: scalars z_1 and vectors $\xi = \left(z_2, ..., z_q\right)^T$. The square of canonical correlation $\nu_1^2\left(\tau, \beta\right)$ of the first component of time series $Z_1\left(t\right)$ with all other components is defined as the maximum eigenvalue of the matrix

$$S_{\xi\xi}^{-1} S_{\xi z_1} S_{z_1 z_1}^{-1} S_{z_1 \xi},$$

where the particular factors are submatrices of the total covariance matrix

$$S_{zz} = M\left(z \cdot z^T\right).$$

The value of the squares of the canonical correlations of each scalar components of a vector time series $Z(t)$ with all other components at the level of detail β in the time window τ when $k=1,...,q$ is equal:

$$v_k^2 = v_k^2(\tau,\beta) \quad (8)$$

Further we extract positive square roots of these quantities. To reduce statistical fluctuations of estimation of canonical correlations due to the decrease in the number of wavelet coefficients with increasing numbers of detail, we introduce an additional averaging over some number of coefficients obtained in the previous windows:

$$\bar{\nu}_k(\tau,\beta) = \sum_{s=1}^{m_\beta} \nu_k(\tau - s + 1,\beta)/m_\beta, \quad m_\beta = 2^\beta \quad (9)$$

The higher the level of detail, the more averaged values (9) are for the previous time windows, that reduce the dependence of the amplitude of statistical fluctuations scatter of estimation (9) on the number of level of detail and makes the variation almost the same for different β. Wavelet coherence measure is:

$$\kappa(\tau,\beta) = \prod_{k=1}^{q} \bar{\nu}_k(\tau,\beta). \quad (10)$$

The value of masure (10) can range from 0 to 1. The higher the value (10), the stronger the aggregate relationship between all processes that are analyzed on the scale corresponding to the number β.

3. Multifractal analysis of the stock indices

Analyzing the financial crisis of the late twentieth century, it can be concluded that the dynamics of deployment of the crisis has some similarities [10-15]. They can be characterized by a universal mechanism of formation and deployment of instability. The global financial crisis of 2008 by all signs can be considered as the global economic crisis, as its influence was felt in the real economy. Moreover, the current financial crises are global and even have geopolitical nature, because they really hurt not only the economic sphere, but also politics. Thus, in the economy all the contradictions of geopolitical relations reflect.

The reasons of financial crises and their drivers mature in different sectors of the economy and related mainly to structural and other changes in the development of the economic system. However, the appropriate situation must be arised for a crisis arise, and drivers should achieve a certain effect. Any reason (driver) of the financial crisis is the crisis of the real economy through an appropriate mechanism. The history of the recent financial crises shows that in many countries the reason of crises was the emergence of new innovative products, technologies, or a sharp increase in demand for real estate. The result is increasing in profitability in the areas of production, causing cross-sectoral flow of capital. Capital and credit flow quickly in high yielding sector, causing a speculative boom and optimism, commercial banks are expanding high-risk operations that weakens the banking system.

According to Shah [15], prior to the global financial crisis, banks were off-loading risky loans onto others by polling their various loans into sellable assets. Although millions can be made from such money-earning loans, they are tied up for decades. Hence, they were turned into securities: the security buyer gets regular payments off those mortgages while the banker off-loads the risk. Once thought to be the greatest financial innovation of the 20th century, securitization, which was meant to reduce risk and increase lending, backfired and triggered the financial crisis.

As a result, world stock markets and large financial institutions collapsed (i.e. job loss leading to social unrest) or were bought out; even governments in the wealthiest nations had to bail out their financial systems [10]. The total that governments (especially in the West) have spent on bailouts is tremendous. According to Bloomberg, 33% or $14.5 trillion of the value of the world's companies have been wiped out by this crisis.

In this paper, we investigate the behavior of the U.S. (DJIA, S&P500, NASDAQ), European (DAX 30, FTSE 100, CAC40) and Asian (SSE, Nikkey 225, Hang Seng, Kospi) stock indices for the period 1988-2013 years [16].

Fig. 1 presents multifractal singularity spectra yield for the corresponding group indices.

Over the spectrum yields, we can conclude that these indicators are multifractal. The generalized Hurst index for indices DJIA and S&P500 is 0,6; for the index NASDAQ – 0,67. For European indices it equals to 0,65 (DAX 30), 0,62 (CAC 40, FTSE 100). For Asian indices generalized Hurst index is 0,65 (Kospi, Nikkey 225), 0,7 (SSE, Hang Seng).

Figure 1: Multifractal spectrum yields for DJIA, Nasdaq, S&P500

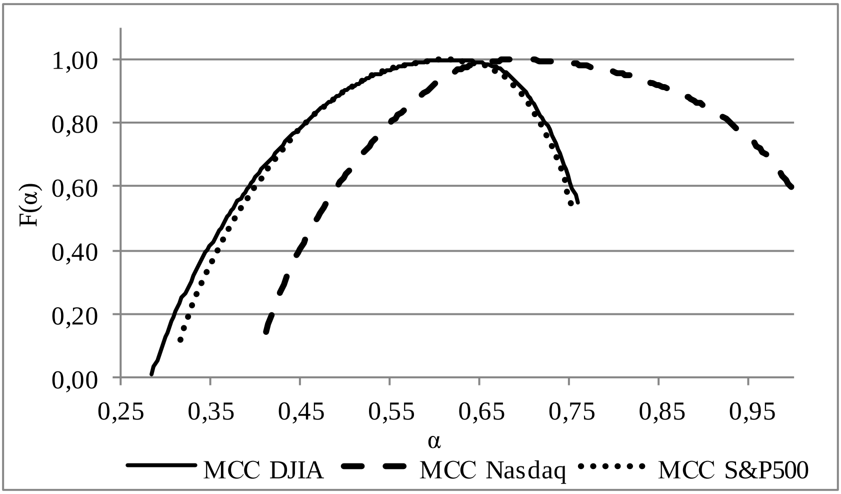

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Figure 2: Multifractal spectrum yields for DAX 30, CAC 40, FTSE 100

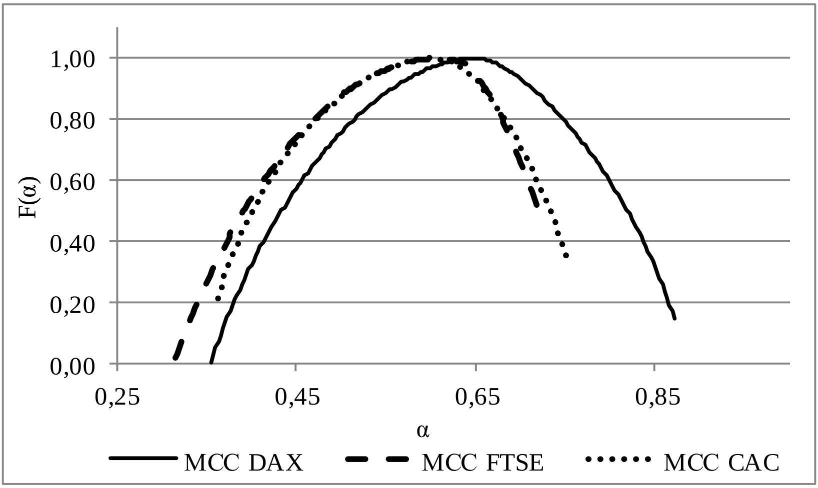

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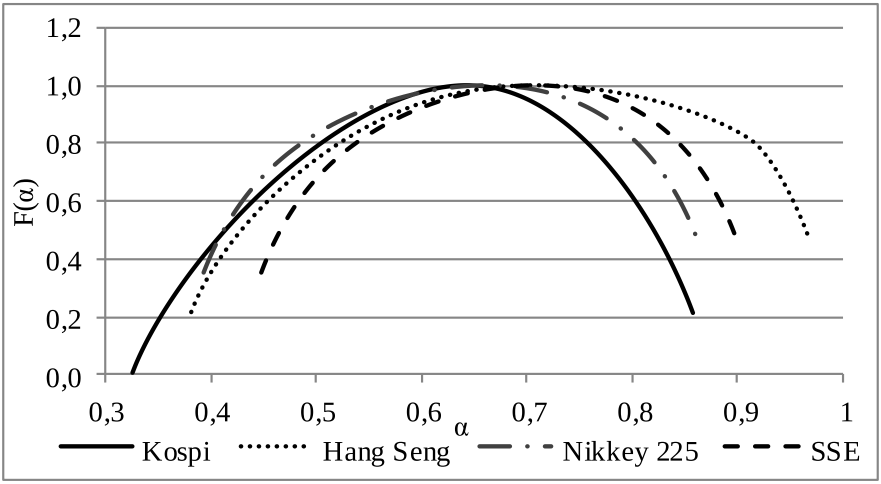
Figure 3: Multifractal spectrum yields for SSE, Nikkey 225, Hang Seng, Kospi



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The comparison of the generalized Hurst index lead to the conclusion that the indices NASDAQ DAX 30 SSE, Hang Seng are comparable to the highest degree of autocorrelation and stability, the dynamics of these indices has more present effect of long memory.

The next step is to study the important parameter - the width of the singularity spectrum, which is a measure of the diversity of random behavior of the signal. Statistics $\Delta\alpha$ of width of the singularity spectrum is an objective measure of synchronization behavior, i.e. the indicator of reducing the chaos before catastrophic events. As the indicator decreases, the probability of adverse events in the stock market is higher.

The dynamics of spectrum width parameter is calculated by the method of moving time window for DJIA DAX 30 CAC 40. The horizontal axis indicates the right end of the time window. Fig. 4 give an idea of the situation on the stock markets during the 1997-2013 years. U.S. mortgage crisis corresponds to a significant drop in the width of the medium range for all three indices

Figure 4: Dynamics of width of the spectrum singularity for DJIA

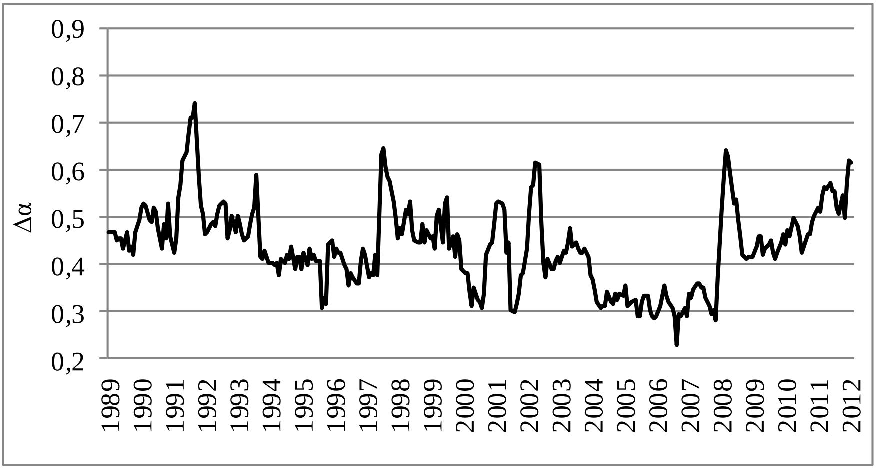

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Fig. 5-6 show fall before the European crisis of 2010-2011.

Figure 5: Dynamics of width of the spectrum singularity for DAX 30

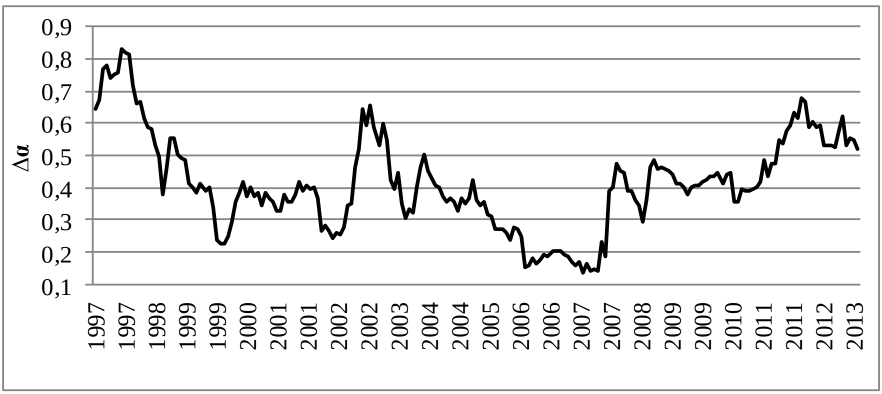

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Figure 6: Dynamics of width of the spectrum singularity for CAC 40

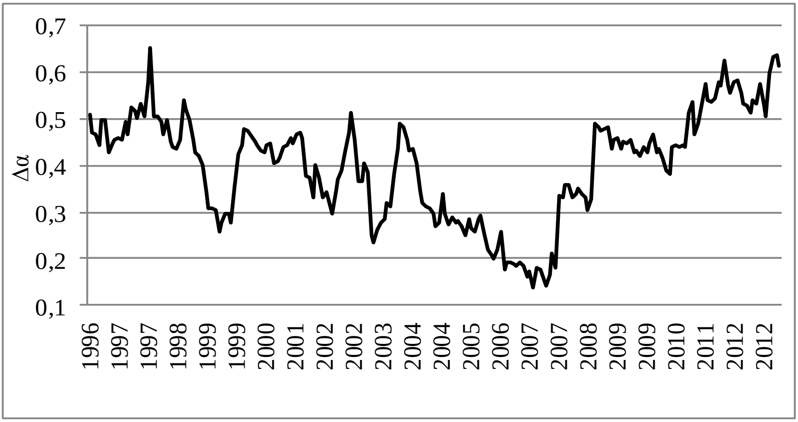

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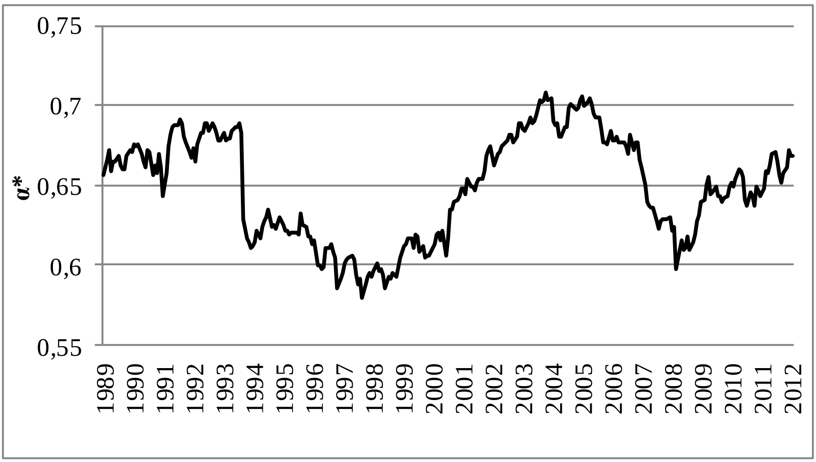
Figure 7: Generalized Hurst index for DJI



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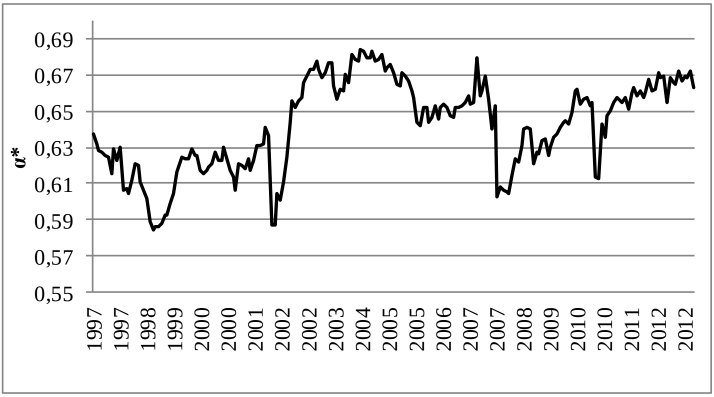
The next step of investigation is to compute the dynamics of the generalized Hurst index. Below we show this indicator for some of the most exemplary indexes. Dynamics of generalized Hurst index of all indices (Fig. 7) has an undulating character.

Figure 8: Generalized Hurst index for DAX 30



Source: authors' own calculations

Significant emissions in the period 2007-2010 correspond to the financial, economic and European debt crises, during which there was a deterioration of all macroeconomic indicators. Measures introduced to stimulate the country's leadership economies and prevent the spread of the financial crisis, failed to keep stock markets from falling. The impact of negative macroeconomic and corporate statistics defined the decline in global stock indices by an average 7-8 % in November 2008.

Figure 9: Generalized Hurst index for FTSE 100

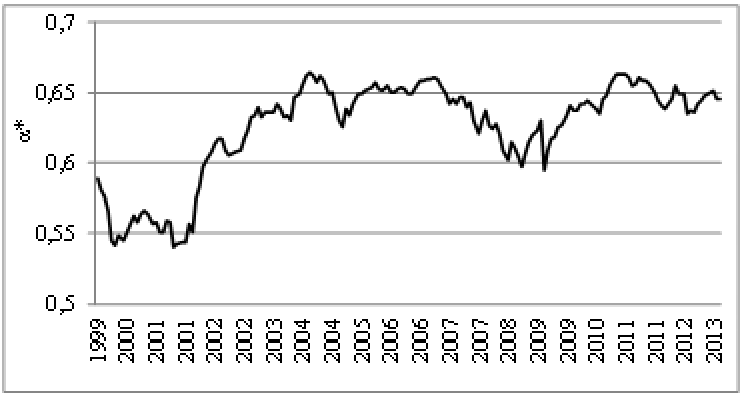


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Figure 10: Generalized Hurst index for CAC 40

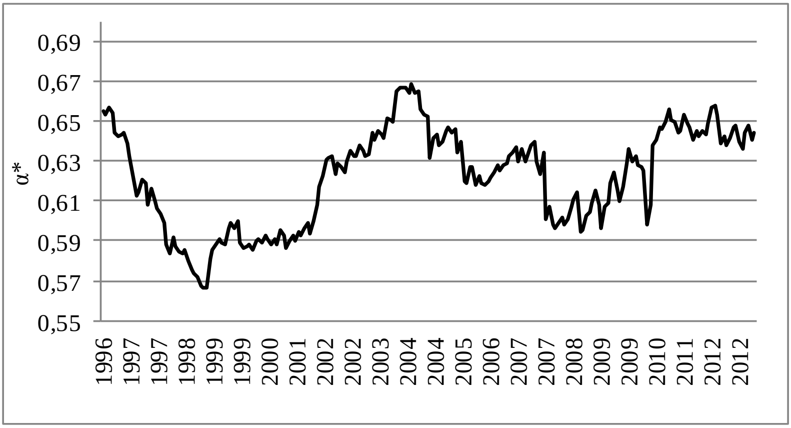

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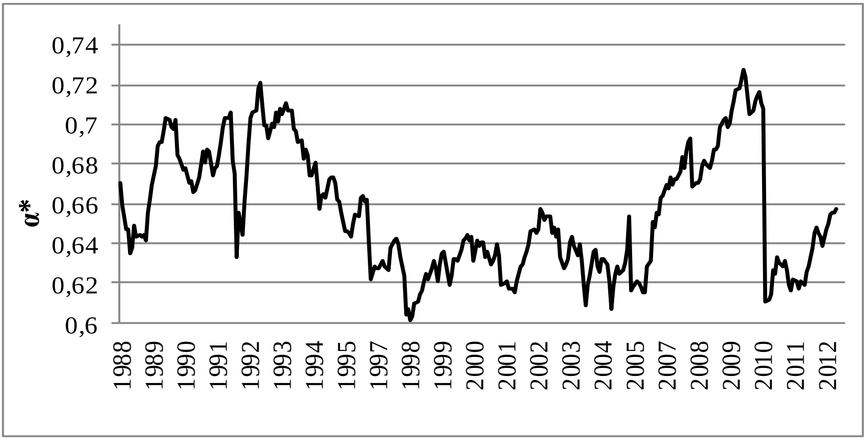
Figure 11: Generalized Hurst index for Nikkey 225



Source: authors' own calculations

The most expressive figure is the generalized Hurst coefficient for Japanese index Nikkey 225, which collapse occurs in 2009. Also, there are fluctuations in the index during the Asian crisis of 1998. One of the reasons of considered Asian crisis is stagnant Japanese economy, fixing national currencies to the U.S. dollar, ineffective programs of the IMF to restore stability in the affected countries. It is also believed that the crisis was initiated by the U.S. to undermine the economic development of Asian countries and eliminate the rapid economic development of China.

First the Asian crisis has covered such countries of south-eastern region as Thailand, Philippines, Indonesia, Malaysia, South Korea, and later spread to Taiwan, Hong Kong, Singapore. Its prerequisites are: the deficit of payments balance, depreciation of the currency, a sharp decline in income of manufacturing companies, reducing the solvency of manufacturing companies, rapid withdrawal of foreign capital during a market panic, reducing liquidity of manufacturing companies. The main internal prerequisite of a financial crisis is uncontrolled growth of credit debt of individual companies.

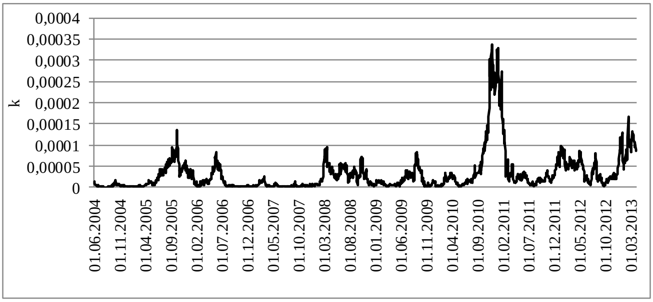
While the Asian financial crisis had very minimal impact on Western economies, the global financial crisis impacted many countries throughout the world, especially U.S, UK and European economies. Although the recovery of countries affected by the Asian financial crisis was relatively fast (i.e. with the help of the U.S and Europe maintaining a level of support), the recovery from the aftermath of the global financial crisis may take a longer time: many Asian countries whose economies where believed to be sufficiently "decoupled" from the Western financial systems gradually become impacted by the implications of the crisis: the rapid growth and wealth creation in many Asian countries (i.e. China) have led to enormous investments in Western countries. Hence, many Asian countries (e.g. China) have seen their stock market suffer and a downward trend in their currency values [10].

4. Coherent analysis of the stock indices

One of the important approaches to the investigation of financial data is to calculate the wavelet coherence measure for investigated time series on the basis of the evaluation of the canonical coherence in the sliding window.

A group of European indices is examined: DAX 30, CAC 40, FTSE 100, WIG 20 and RTS. The length of the window was equal to 365 days, calculations were carried out at four levels of detail. The first level of detail corresponds to a time scale variations between 2 to 4 days, the second - between 4 to 8 days, the third - between 8 to 16 days, and the fourth - between 16 to 32 days. In Fig.12-14 it is shown the dynamics of wavelet coherence measures for studied indices at appropriate levels of wavelet decomposition detail.

Figure 12: Wavelet coherence measure of 1 level detail for European indices.

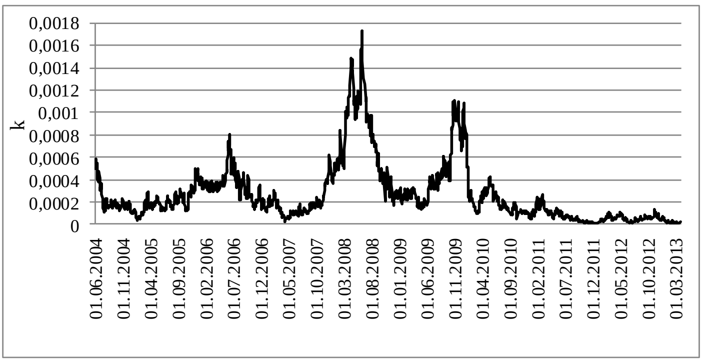


Source: authors' own calculations

In figure 12 a sharp growth rate in 2010 is marked, that is in line with the European debt crisis at this period. In the first quarter of 2008 there is an increase of 10 times in measure that is a warning of dange, which resulted in a financial crisis.

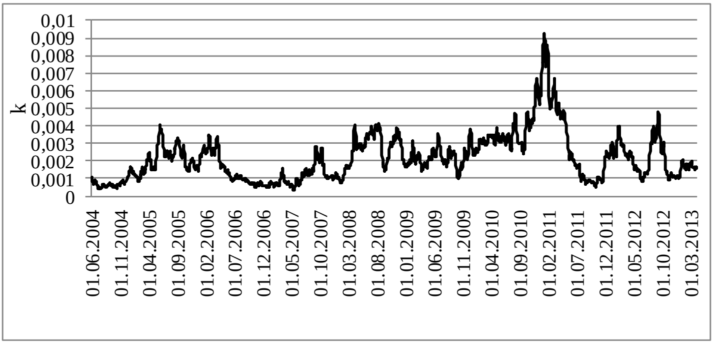
On the second level of detail (Fig. 13) of wavelet decomposition, the situation is somewhat different. A distinct character of growth measures is observed before the crisis of 2008. At the end of 2007, the degree of coherence is growing rapidly and in the second quarter of 2008 takes its maximum value. Another release is presented in Fig.13 at the end of 2009, which is the precursor to the European crisis and a signal on the use of effective anti-crisis measures.

Figure 13: Wavelet coherence measure of 2 level detail for European indices.



Source: authors' own calculations

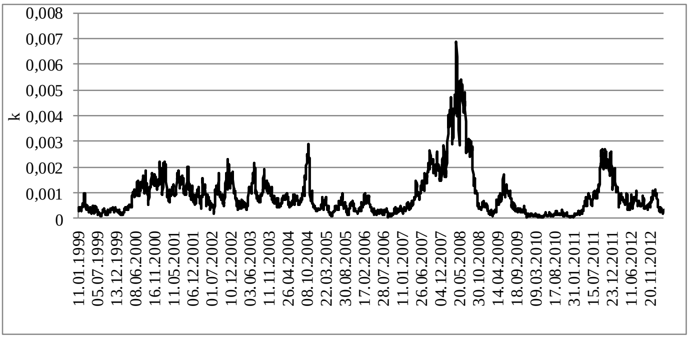
Figure 14: Wavelet coherence measure of 3 level detail for European indices.



Source: authors' own calculations

In Fig. 14 it can be observed a variation of index since the beginning of 2007, the dynamics is somewhat similar to the situation in Fig. 12. For comparison, consider the wavelet coherence for the 2nd level of detail for the Asian indices.

Figure 15: Wavelet coherence measure of 2 level detail for Asian indices



Source: authors' own calculations

The dynamics of parameters in Fig. 15 is characterized by fluctuations during 2000-2004, which grows in the maximum release in May 2008, on the eve of the financial crisis.

The nature of the dynamics of coherent measures of 1 and 3 levels of detail are similar. It manifests itself in increasing measure of coherence in

2004-2005, the reason is the unstable situation in the Asian market due to local stress value.

5. **Conclusions**

Analyzing the financial crisis of 2008-2009, it can be argued that all calculated values were warnings of danger. The global financial crisis of 2008 was reflected in the September - October 2008 in the form of a sharp deterioration in key economic indicators in most developed countries. The crisis has become global nature and caused a sharp decline in output, consumer demand, commodity prices and rising unemployment. This was reflected in a significant drop in the values of stock indexes.

During the European debt crisis that began in 2010, the values of multifractal indexes of UK, France, and Germany dramatically changed. There has been a fall of generalized Hurst index and reduce the width of the media spectrum. The crisis was caused by increasing levels of public debt of European countries, the growing budget deficit, the loss of competitiveness of some European countries and the lack of flexibility in the monetary policy.

The dynamics of wavelet coherence measure for all countries and of all three levels of detail presents an impressive picture of the deployment of the financial crisis of 2008-2009 and the European debt crisis of 2010-2013. The synchronization of behavior of indices yields are observed before the crisis, while the characteristic differences of these crises are appreciable at different frequencies.

6. References

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