

## INDICATIONS OF CHAOTIC BEHAVIOUR IN USD/EUR EXCHANGE RATE

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### **Abstract:**

*Determining if a process is or not chaotic may supply valuable information about how to deal with that process. The predictability of a time series depends on his nature. In the case of chaotic time series, the maximal Lyapunov exponent determines the prediction horizon. Because there is no single test that completely identifies chaos, it is recommended to perform various tests that provide indices of chaos. Several tests have been applied on the USD/EUR exchange rate revealing some interesting evidences of chaos.*

**Keywords:** Chaos tests, Time Series, Exchange rate.

**JEL Classification:** C02, F31, F37.

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### **1. Introduction**

Chaotic systems are, in fact, complex deterministic systems with a large number of variables that influence the evolution of a process, making it impossible for humans to simulate it and, therefore, inducing the idea of

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unpredictability. This, also, makes it impossible to determine the initial state of the system knowing just the final state (Georgescu, 2012).

Most processes and systems found in nature involve the interaction of many factors, which allows us to catalogue them as chaotic systems. Thus, chaos is met in: solar system dynamics, evolution of populations, weather, chemical reactions, etc.

Also, economy itself can be seen as a chaotic system, a factor that brings a huge number of variables is people's direct involvement. The chaos from complex systems is known as deterministic chaos.

For chaotic systems, the fact that they are deterministic does not make them predictable. However, the predictive power in the case of chaotic systems can be improved and illustrate this with weather system for which predictions for short periods have reached a very good accuracy (Ciobanu, 2012).

We must emphasize the fact that the emergence and development of chaos theory could not have taken place before the invention of computers as simulation of complex systems with many variables could not have been done without their help.

An important feature of chaotic systems is Sensitive Dependence on Initial Conditions (SDIC). This tells us that two initially close trajectories depart exponentially in a finite number of iterations, sometimes very quickly. In such a system, prediction is impossible, except, maybe, for very short-term prediction (Goldsmith, 2009). The most frequent tool used for identifying these processes from dynamical systems theory or experimental series is Lyapunov characteristic exponent (LCE).

## **2. Tests of chaos done on the time series of exchange USD/EUR rate**

I used the time series of USD/EUR exchange rate. Historical values of exchange rate are available for download on the site of the European Central Bank at [http://sdw.ecb.europa.eu/browse/Selection.do?DATASET=0&sf11=4&FREQ=D&sf13=4&CURRENCY=USD&node=2018794&SERIES\\_KEY=120.EXR.D.USD.EUR.SP00.A](http://sdw.ecb.europa.eu/browse/Selection.do?DATASET=0&sf11=4&FREQ=D&sf13=4&CURRENCY=USD&node=2018794&SERIES_KEY=120.EXR.D.USD.EUR.SP00.A).

The considered time series contains 3656 records during 04.01.1999 - 10.01.2013 and consists of USD/EUR exchange rate quotations established by the European Central Bank for week days.

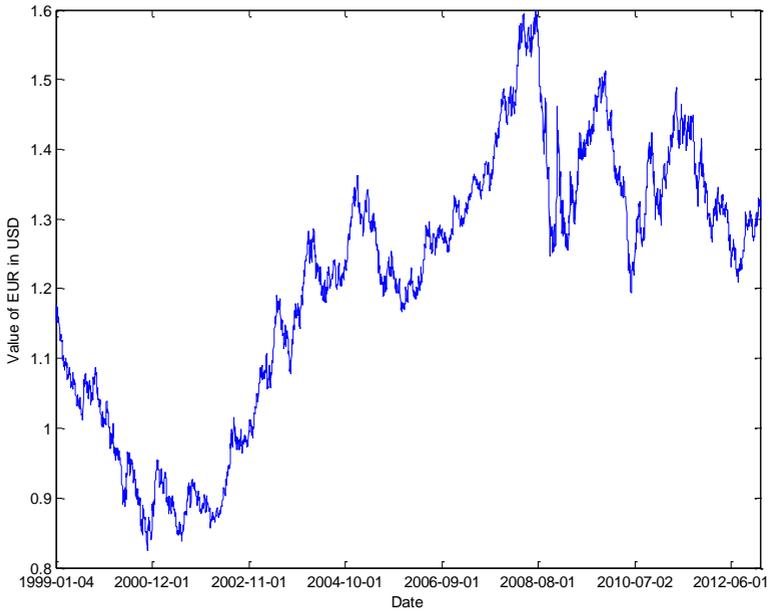
Missing data, approximately 50 records, was replaced by the

mean of neighboring values.

After preprocessing we obtained a time series with a mean of 1.2106 and a standard deviation of 0,1897.

For modeling and simulations we used MATLAB (R2011 a) and tstool Toolbox (time series tools).

**Figure 1. The evolution of USD/EUR exchange rate over time.**



From the graphical representation Figure 1. we can see that, in the case of the USD/EUR exchange rate we deal with strongly nonlinear process. Nonlinearity does not imply chaos, but all chaotic processes are nonlinear.

The working mode for highlighting chaos in time series is sinking them into a multidimensional space.

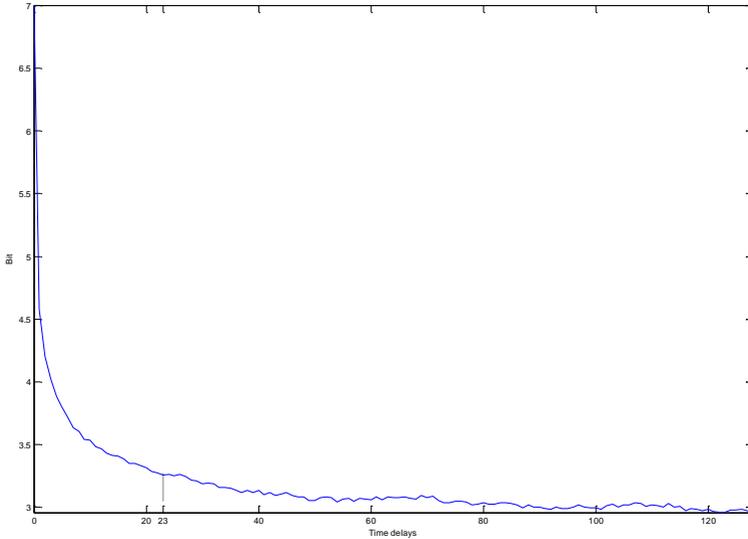
Transition from one-dimensional time series to the corresponding  $d$ -dimensional series in state space is done using Takens theorem (Takens, 1981).

We embed one-dimensional series in a  $d$ -dimensional space by building vectors of length  $d$  as follows:

$$x_t^d = (x_t, x_{t+\tau}, \dots, x_{t+\tau(d-1)}), \quad t = 1, 2, \dots, N - \tau(d-1),$$

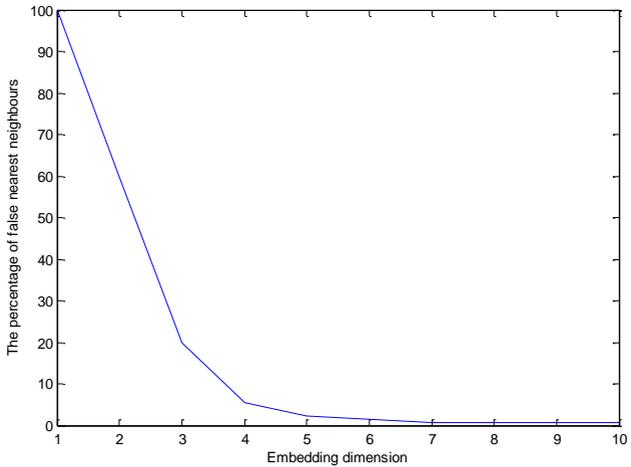
where  $\tau$  is the number of time delays.

**Figure 2. Auto-mutual information function.**



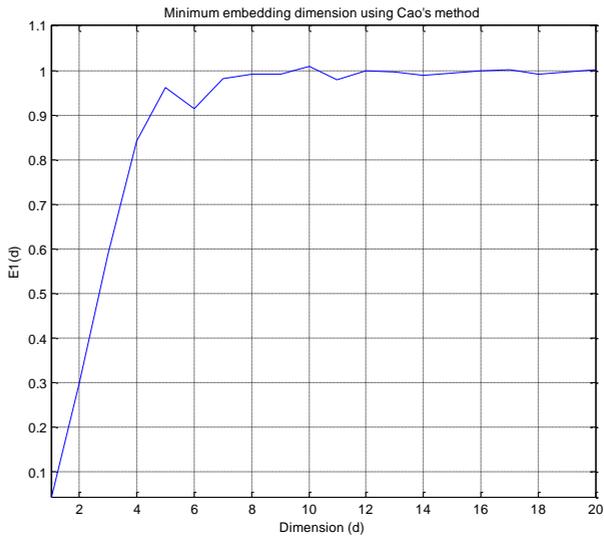
Determining a suitable time delay is achieved by building auto-mutual information function and finding his first local minimum. We have conducted several simulations and the obtained value for the delay time was  $\tau=23$ .

**Figure 3. Minimum embedding dimension obtained with false nearest neighbors method.**



Simulations have indicated in the case of using the false nearest neighbor method a minimum embedding dimension between 6 and 8 (Figure 3).

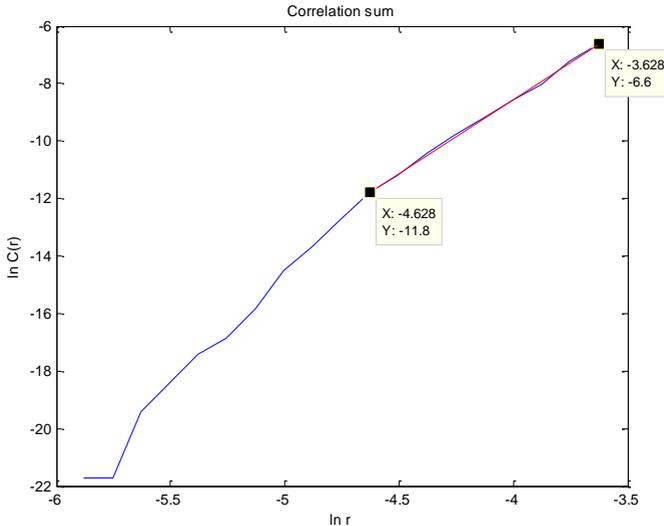
**Figure 4. Minimum embedding dimension obtained with Cao method.**



And, if using Cao method for determining the embedding dimension, we obtained values between 6 and 8 (Figure 4). We decided that minimum embedding dimension is 7, the most often value indicated by tests.

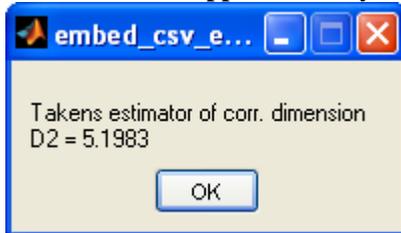
We used the two values time delay  $\tau=23$  and minimum embedding dimension  $d=7$  to determine multidimensional series.

**Figure 5. Determining the correlation dimension.**

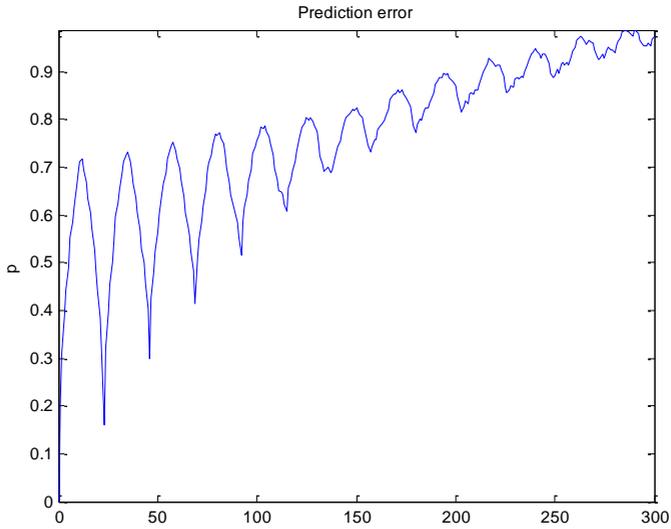


The linear portion from graphical representation (Figure 5) has a slope of about 5.2. The Takens estimator for the correlation dimension was 5.1983, results obtained with an instrument from the `tstool` toolbox (Figure 6). The non integer value is an indication of chaos.

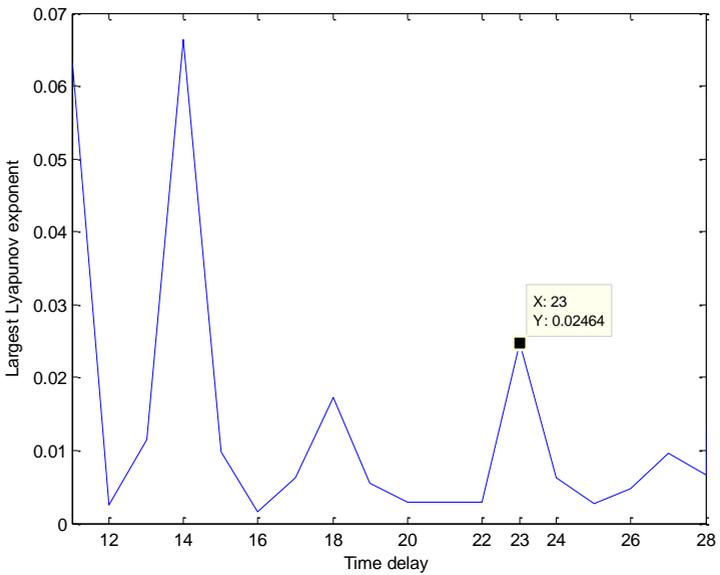
**Figure 2. The correlation dimension approximate by the Takens estimator.**



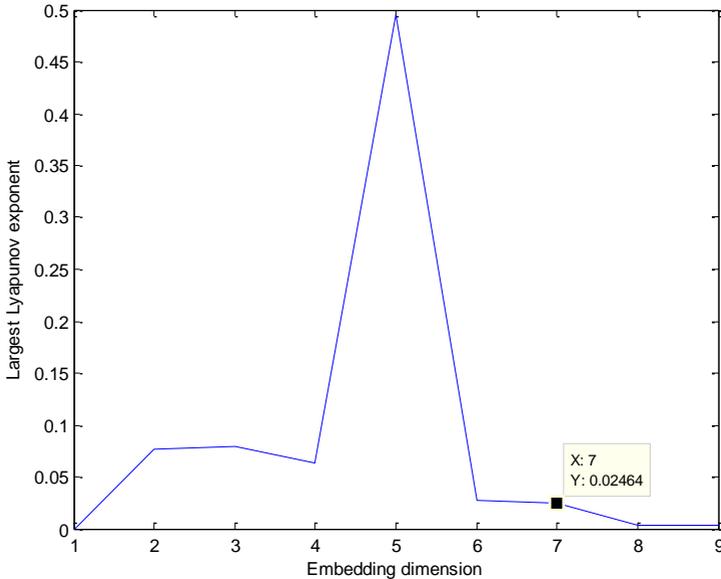
**Figure 3. The evolution of the prediction error.**



**Figure 8. The largest Lyapunov exponent versus time delay.**



**Figure 9. The largest Lyapunov exponent versus embedding dimension.**



The graphical representations from Figure 8. and Figure 9. show the evolution of the largest Lyapunov exponent depending on the time delay, respectively dimension of embedding space.

Positive values for the largest Lyapunov exponent indicate a chaotic nature for the time series of the exchange EUR-USD rate.

### 3. Conclusions

In economy the majority of historical data are available as time series. Detecting chaotic nature of the processes that have provided such data is not an easy task, because there is still no way to clearly specify the existence of chaos. Another restriction is the relatively small number of observations that allows us only to issue certain assumptions about the phenomenon studied and to determine estimates of chaos indicators such as largest Lyapunov exponent.

Thus in this uncertainty we can only try to highlight as many aspects that allow us cataloging the process as a chaotic one (Georgescu, 2012).

Due to these weaknesses and others, such as the difficulty to distinguish between deterministic chaos and noise and limited predictions to just a few steps, economists have lost the enthusiasm displayed upon discovery of chaos theory.

However, there are ideals such as guiding the economy with small impulses applied at appropriate times, aimed at by theorists in economics that could be possible using models based on chaos theory.

In the case of the time series of the exchange rate between the euro and dollar, simulations indicate the presence of chaos. With a largest Lyapunov exponent of about 0.025, theoretically acceptable predictions are possible for a number of about 40 steps. Thus, the problem of determining the model that simulates reasonably well the time series of the exchange rate remains open so that predictions for first steps to be within acceptable error margin.

Determination of chaotic behavior is important from this regard to establish a correct prediction horizon.

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